Monitoring Costs and the Management of Common-Pool-Resources

WORKING PAPER

repository: https://fossil.gpfreitas.net/fishing/index.cgi
checkout: 76b32d9fadd04fde253c39d9b421d2ff005c963f 2013-05-10 05:25:48 UTC

Guilherme Freitas *

May 10, 2013

Abstract

We lay down a model of a fishery and analyze the outcomes of a program of individual tradable quotas (ITQs) when quota enforcement is costly and imperfect. In this setting, decisions about enforcement level should not be dissociated from other design decisions—like the total quota available or its initial distribution. To support those design decisions, we provide an extensive analysis of ITQ equilibria and full comparative statics for steady-state equilibria. To the best of our knowledge, this is the first time this analysis is carried out.

We also provide an extension of the full-compliance result that states that an ITQ program leads to an efficient use of the fish stock. Relaxing the Assumption of full compliance, we present a principal-agent model where the principal is a fishery owner and the agents are the fishermen. The principal chooses how to allocate quota among the fishermen and how much to invest in monitoring to set the enforcement level. Agents choose how much fish to catch in face of their quota and the enforcement level. We show that, while the first-best outcome is not incentive-compatible, second-best outcomes can be implemented by an ITQ program if, and generically only if the expected violation fines depend on catch and quota only through absolute violations.

Finally, we establish sufficient conditions for fishermen's preferences over small changes in enforcement to be single-peaked. We emphasize that even though the distribution of quota endowments does not affect the attained ITQ equilibrium directly, it may affect outcomes indirectly if fishermen can influence the process that sets the cap or enforcement levels—with or without quota trading.

*Many thanks to John Ledyard, Jean-Laurent Rosenthal and Federico Echenique for helpful discussions. I am also grateful to my colleagues at the graduate student seminar at Caltech. Any errors are my own. Financial support from the California Institute of Technology is gratefully acknowledged.

1 Introduction

Since the seminal papers of Dales (1968), Montgomery (1972) (in the context of pollution) and Moloney and Pearse (1979) (in the context of fisheries), individual tradable quotas (ITQs) have become a very popular tool in the management of common-pool resources (CPRs), attracting a lot of attention not only in academia, but also in government and industry.[1] To have an idea of the impact that an improvement on the management of such programs can have in fisheries worldwide, we point out that, according to Bonzon, McIlwain, Strauss, and Van Leuvan (2010)), one out of every five coastal countries are using some form of catch-share regime to manage over 850 species of fish, and the adoption of ITQ programs continues to increase.

ITQs became so popular not only because they can prevent the collapse of a natural resource, but also because they allow the resource stock to be exploited efficiently without requiring centralized knowledge of information about individual agents' private information. The argument is by now standard and featured in standard textbooks:[2] the cap will avoid over-exploitation[3] of the resource, while the quota market will allow the more efficient producers to buy the production rights from the less efficient producers and produce the target output at a lower cost.[4] There are other theoretical arguments in favor of ITQs, but those are beyond the scope of this paper.[5]

Beyond theoretical predictions, there is strong evidence that cap-and-trade programs often perform well in practice: Costello, Gaines, and Lynham (2008) compiled a global database on 11,135 fisheries from 1950 to 2003 and concluded that the fraction of ITQ-managed fisheries that collapsed was about half that of non-ITQ fisheries.[6] ITQs also avoids the race for fish and the consequent rent dissipation. See Knapp and Murphy (2010) for an experimental argument and Bonzon, McIlwain, Strauss, and Van Leuvan (2010) for a large number of references on the effects of ITQs.

One point where both academics and managers agree is that no cap-and-trade program can

[1] See Freeman and Kolstad (2007), Grafton, Arnason, Bjorndal, Campbell, Campbell, Clark, Connor, Dupont, Hannesson, Hilborn, Kirkley, (2006), and Bonzon, McIlwain, Strauss, and Van Leuvan (2010).

[2] See for example Perman, Ma, Common, Maddison, and Mcgilvray (2012), Clark (2010), Tietenberg and Lewis (2008), and Conrad (2010).

[3] Of course, this depends on the growth rate of the resource and on the discount factor of the quota-holder, as exemplified by the case of the Antarctic blue whale fishery Clark (1973). For an argument that this should not be a problem in general, see Grafton, Kompas, and Hilborn (2007).

[4] That ITQ programs stand on those two pillars—the cap and the market— is the reason why such programs are also known as *cap-and-trade* policies.

[5] See the aforementioned textbooks or Bonzon, McIlwain, Strauss, and Van Leuvan (2010).

[6] Their definition of collapse is taken from Worm, Barbier, Beaumont, Duffy, Folke, Halpern, Jackson, Lotze, Micheli, Palumbi, Sala, Selkoe, Stac (2006): a fishery collapses in year t if the harvest in that year is less than 10% of the maximum recorded harvest up to that year.

work without adequate monitoring and quota enforcement. For example, Copes (1986) explains how quota violations led to the abandonment of the cap-and-trade program at the Bay of Fundy herring fishery.[7] However, little theoretical work has been done to understand how costly and imperfect enforcement affect outcomes. The work that is closest to ours was done by Malik (1990), in the simpler context of air pollution, and Hatcher (2005) and Chavez and Salgado (2005) in the context of fishing.[8] However, none of those models take into account the stock dynamics of the resource or technological differences among fishermen. Without the first, steady-state analysis is not possible, and without the second, we cannot examine how preferences for the cap or enforcement levels may differ across economic agents.

Our point of departure is a classic result in the cap-and-trade literature with perfect and costless monitoring: the outcomes of an optimal.[9] command-and-control policy can be achieved by setting a total output cap and a market for quotas. That is the well-known ***efficiency of ITQ markets*** An important question is whether or not there is a similar result in the case of imperfect enforcement. If violations were perfectly and costlessly observable, the problem would be easy: set fines for violation high enough and nobody will violate their quota in equilibrium, which we know is the first-best outcome. In reality it is costly to observe quota violations, and therefore it may not be socially optimal to have zero quota violations. This new optimum is what we call the *second-best* outcome, a concept we make precise in Section 4. In the simpler setting of air pollution, Malik (1990) showed that for the attainment of a second best, it is necessary that expected violations depend only on absolute violations instead of, say, violations as a proportion of quota, or magnitude of the catch. We provide a similar and more complete characterization in the more complicated case of renewable resources: it is sufficient and *generically* necessary that expected fines depend only on absolute violations for a second best to be attainable by an ITQ program. While building this result, we uncover the fact that optimal cap and enforcement levels imply a positive amount of quota violations.

We go on to build, from the bottom up, the equilibrium notion we will focus on: the stable, steady-state equilibrium. The most important concept we need before the aforementioned equilibrium is that of a *temporary equilibrium given a stock level s*, which is simply a competitive market equilibrium when the stock level is s. We present full comparative statics for both types of equilibria, in particular how the steady-state equilibrium changes with respect to changes in the cap and enforcement levels. Along the way, we show that ITQ equilibria have the following *all-or-nothing* property: either nobody violates their quota, or everyone violates their quota.

[7] He also discusses other hurdles that can get in the way of the well functioning of a cap-and-trade program.

[8] See Montero (2004) for some related empirical work.

[9] In the sense of maximizing the total industry profit.

Note that while Chavez and Salgado (2005) provide *some* comparative statics for what we call *temporary equilibrium*, they do not have a model for the replenishment of the natural resource, and thus cannot present *steady-state* comparative statics. As we will see, steady-state analysis is a more delicate matter than temporary equilibrium analysis because we may have multiple equilibria, and an equilibrium may be degenerate or unstable. The work of Hatcher (2005) also does not touch on steady-state issues and makes the decision to comply with or violate quota exogenous, thereby leading to a result that is different from ours: that if expected violation fines are a function of absolute violations *as a fraction of quota held*, then the quota price in a compliant market may be lower than in an otherwise non-compliant market.

Finally, with imperfect and costly enforcement, the question of how much enforcement becomes central, as well as the complementary effects of raising enforcement or lowering the cap. Monitoring affects all fishermen irrespective of who paid for it, and therefore fishermen may have an incentive to free-ride on the contributions of others. Indeed, it is not hard to show that letting enforcement be paid solely by voluntary contributions will lead to no enforcement at all. There is a more basic problem however: even if no fisherman has to pay for monitoring costs, one fisherman might want more monitoring, while another might want less. The reason is simple: all else equal, buyers in the quota market want a low quota price, and sellers want a high quota price. Intuitively (and shown in Theorem 7) a higher level of monitoring leads to higher quota prices. At an even more fundamental level, agents may disagree on the level of monitoring even when no quota trade is allowed. That is because increased monitoring leads on one hand to higher stock levels and thus lower costs for fishermen, but also to higher violation fines for the same fishermen. For some fishermen, the cost decrease might offset the steeper violation fines; for others, it might not. In light of the ambiguity outlined above it is hard to obtain strong conclusions about collective preferences over monitoring. We conclude the paper with a first-step in that direction with a local result (theorem 8), saying that under certain conditions —including no quota trade, or no wealth effects in the quota market— if a given boat wants slightly more monitoring, then all larger boats will want slightly more monitoring.

We now proceed to the model in Section 2. There we will define the primitives of the model and our core Assumptions. In Section 3 we define our notion of ITQ equilibrium. Section 4 presents the single-owner problem and some of the properties of a solution. In that Section we establish our first main result: the solution to the single-owner problem can be implemented by an ITQ program if and generically only if expected violation fines depend only on absolute violations. We go on to perform a detailed analysis of equilibria in Section 5. We split the analysis in three layers: individual optimal behavior, temporary equilibrium (a competitive equilibrium given a fixed stock level). and steady-state equilibrium. This conceptual organization allows us to analyze existence, multiplicity, regularity and stability of equilibria in a convenient and intuitive way. In this Section we also establish our second main result: in any temporary equilibrium either nobody violates their quota or everyone does. Restricting ourselves to regular, stable equilibria, we then show how equilibrium points change when the monitoring expenditure M and the $\mathcal{TAC}$ level suffer small changes. We close the paper with a brief analysis of a fishery with larger boats and smaller boats (multiple types). We provide some comparisons between the equilibrium decisions of different fishermen, and explain how endowments may affect equilibria if fishermen have any influence on the design variables that determine the enforcement levels or the cap. Precisely, we show that if endowments are such that changes in the design variables cause no changes in wealth, then support for more monitoring will come from larger boats, if from anyone at all. A brief conclusion follows, where we provide final remarks on our results and directions for future research.

2 The Model

Let $\mathcal{I}$ be a set of $n \in \mathbb{N}$ fishermen. Each fisherman produces an amount $y_i \geqslant 0$ of fish (referred to i's ***production***, ***output*** or ***catch***) using a ***technology*** $\theta_i \in \Theta$. The ***cost*** $c(y_i, s, \theta_i) \in \mathbb{R}$ incurred by agent i depends not only on his catch y_i and technology θ_i but also on the ***stock of fish*** $s \geqslant 0$. The fishermen can sell their output in a competitive market where the ***price of fish*** is $p > 0$. Their objective is to choose a level output y_i such that their profits are maximized.

The stock of fish is governed by the law

$$s' = g(s) - Y(s) \tag{1}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a growth function and $Y(s)$ is the ***total catch***, that is, the sum of each fisherman's i' catch $y_i(s)$ when the stock of fish is s. We will restrict attention to ***steady-state*** outcomes, that is, those where $s' = 0$.

Our goal is to study the performance of this industry under a ***cap-and-trade*** program —also known as a program of ***individual tradable quotas*** (ITQs)— when monitoring is imperfect. To that end, we will introduce a regulator in the model. She[10] has three ***regulatory instruments***: the cap, the initial quota endowments, and the level of monitoring (we will also call those ***design variables***). In more detail, the regulator sets a ***total allowable catch*** $\mathcal{TAC} \geqslant 0$, and allocates ***initial endowments of quota*** $\omega_i \geqslant 0$ to each fisherman i such that $\sum_i \omega_i = \mathcal{TAC}$. She also determines the *monitoring* expenditure $M \geqslant 0$. The fishermen then observe the regulator's decisions, the stock level s and the quota price q and make their choices: how much quota to

[10] Simple convention: fishermen are male, regulator is female.

buy and how many fish to catch. The regulator's instruments $(M, \mathcal{TAC})$ affect the fishermen via a quota market where the ***price of quota*** will be denoted $q \geqslant 0$ and through ***quota violation fines*** $\phi(M, y_i, w_i) \geqslant 0$ charged to agent i for catching y_i while holding quota w_i. In that setting, a fisherman's ***profit*** is given by

$$\pi_i = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i) - \phi(M, y_i, w_i). \tag{2}$$

2.1 Notation, Conventions, and Assumptions

Unless otherwise noted, we write $y = (y_1, \dots, y_n)$, $w = (w_1, \dots, w_n)$, $\omega = (\omega_1, \dots, \omega_n)$, and capital letters denote aggregates, so $Y = \sum_{i \in \mathcal{I}} y_i$. When we make a statement involving y_i, w_i, θ_i without specifying which agent i we are talking about, it is to be understood that the statement is true for any fixed $i \in \mathcal{I}$ that is consistent with the given context.

We denote the partial derivative of a function $f : \mathbb{R}^n \to \mathbb{R}$ with respect to it's i-th argument as $D_i f$, and we define $D_{ij} f \equiv D_i(D_j f)$. When we want to differentiate a mathematical expression h with respect to a certain variable x we write $D_x h$. *Example:* we use $D_1 c$ for marginal cost, and $D_M \pi_i^*$ for the derivative of a value function π_i^* with respect to the parameter M.

When defining monitoring functions ϕ, it is useful to have a shorthand notation for the positive part map which we will write as $[x]^+ = \max\{0, x\}$.

We will always maintain the following Assumptions on the functions c, g, and ϕ.

Assumption 1. We define agent i's technology $\theta_i \in \Theta$ as his boat capacity, and assume that $c(y_i, s, \theta_i) = \infty$ whenever $y_i \geqslant \theta_i$. Furthermore, we assume that where $y_i < \theta_i$, c is twice differentiable, strictly increasing, and strictly convex in output y, and decreasing in stock s. Finally, we assume that marginal costs $D_1 c$ are decreasing in both stock s and boat size θ. The convexity Assumption is standard, and will contribute to making the individual optimization problems (4) convex; the Assumption of $D_1 c$ decreasing in θ expresses the heterogeneity among agents, namely that larger boats, with higher fixed cost, have lower marginal costs.

Assumption 2. The growth function $s \mapsto g(s)$ is concave, satisfies $g(0) = 0$, $g(K) = 0$, and $g(s) > 0$ when $s \in (0, K)$, where $K > 0$ is the maximum stock of fish that can be sustained by that environment's resources in a steady state, known as the ***carrying capacity*** of that environment. We also assume that the maximum of g —known as the ***maximum sustainable yield*** (MSY)— is attained at a unique stock level $sMSY > 0$.

Assumption 3. The ***monitoring function*** is defined by

$$\phi^+(M, y_i, w_i) = \max\{0, \phi(M, y_i, w_i)\} \tag{3}$$

where $(M, y_i, w_i) \mapsto \phi(M, y_i, w_i)$ is twice continuously differentiable, increasing in M, increasing and convex in y_i, decreasing and strictly convex in w_i. equal to zero whenever $M = 0$ or $y_i = w_i$.

The easiest way to think of ϕ is to assume it has the form

$$\phi(M, y_i, w_i) = \rho(M) v(y_i, w_i)$$

where $\rho(M)$ is the probability of being audited and $v(y_i, w_i)$ is some penalty function for violations. *It should be clear that we are implicitly making the important implicit Assumption that fishermen are risk neutral.*

3 Equilibrium with Individual Transferable Quotas

From the regulator's point of view, her choice of $(M, \mathcal{TAC}, \omega)$ leads to an ***outcome*** (y, w, q, s) (all entries being non negative). We assume the fishermen behave *competitively* —that is, they do not take into account the impact of their catch on the stock— and we study equilibrium outcomes.

An (ITQ) ***equilibrium*** is an outcome (y, w, q, s) that satisfies the following three conditions.

1. *Fishermen maximize profits.* Given the regulator's choice of $M, \mathcal{TAC}, \omega$, the price of fish p, the price of quota q and the state s, every fisherman's $i \in \mathcal{I}$ choice of y_i, w_i maximizes profits (2), i.e., it solves

$$\underset{y_i, w_i}{\texttt{maximize}} \quad \pi_i \quad \texttt{subject to} \quad y_i \geqslant 0,\ w_i \geqslant 0. \tag{4}$$

2. *The quota market clears.* That is,

$$\sum_{i \in \mathcal{I}} w_i = \mathcal{TAC}. \tag{5}$$

3. *The fish stock is at a steady state.* In other words, the total amount of fish caught is the same as the amount by which the stock grows:

$$Y = g(s) \tag{6}$$

An outcome that satisfies conditions 1 and 2 above is called a ***temporary equilibrium***.

The set of equilibria corresponds to the set of non negative solutions to a system of equations. Recall that the individual problems (4) are convex, and all the constraints are affine. Therefore, we can write the Lagrangian of the problem of agent i as

$$\mathcal{L}_i(y_i, \mu_i^y, w_i, \mu_i^w) = p y_i - c(y_i, s, \theta_i) - q(w_i - \omega_i) - \phi(M, y_i, w_i) + \mu_i^y y_i + \mu_i^w w_i \tag{7}$$

and conclude that the following conditions are necessary and sufficient for an outcome (y, w, q, s) with $y_i \neq w_i$ for all $i \in \mathcal{I}$ (that is the smooth case, we will deal with the nonsmooth case shortly) to be an equilibrium: for every $i \in \mathcal{I}$ there exists $\mu_i^y \geqslant 0$ and $\mu_i^w \geqslant 0$ such that the system of equations below is satisfied. *Notation:* we write c_i for $c(y_i, s, \theta_i)$ and ϕ_i for $\phi(M, y_i, w_i)$.

$$p - D_1 c_i - D_2 \phi_i + \mu_i^y = 0 \qquad i = 1, \ldots, n \tag{8}$$

$$-q - D_3 \phi_i + \mu_i^w = 0 \qquad i = 1, \ldots, n \tag{9}$$

$$\mu_i^y y_i = 0 \qquad i = 1, \ldots, n \tag{10}$$

$$\mu_i^w w_i = 0 \qquad i = 1, \ldots, n \tag{11}$$

$$\sum_{i=1}^{n} w_i - \mathcal{TAC} = 0 \tag{12}$$

$$\sum_{i=1}^{n} y_i - g(s) = 0 \tag{13}$$

Note this is a square system: $4n + 2$ variables and equations. The first $4n$ equations indicate individual optimal behavior; the last two equations say that the quota market clears and the environment is at a steady state, respectively. The first $4n + 1$ equations characterize a temporary equilibrium.

When $y_i = w_i$ for some $i \in \mathcal{I}$, the monitoring function ϕ need not be differentiable and thus the equations in (8, 9) are replaced by the following conditions (see Lemma 11 in page 30 for more details). For every i such that $y_i = w_i$ there exists $\mu_i^y \geqslant 0$, $\mu_i^w \geqslant 0$, *and* $\alpha_i \in [0, 1]$ such that $y_i \mu_i^y = 0$, $w_i \mu_i^w = 0$, and

$$p - D_1 c_i - \alpha_i D_2 \phi_i + \mu_i^y = 0 \tag{14}$$

$$-q - \alpha_i D_3 \phi_i + \mu_i^w = 0 \tag{15}$$

where $D_2 \phi_i$ and $D_3 \phi_i$ are the lateral derivatives defined earlier.

Remark 1. First, because fishermen have quasilinear preferences, the regulator's choice of quota endowments $\omega \in \mathbb{R}_+^n$ does not affect the equilibrium outcome, only the equilibrium profits. Second, the regulator's choice of the $\mathcal{TAC}$ enters the agents decision problem only *indirectly*; it affects equilibrium solely through the market-clearing condition (5). Third, observe that condition 1 above is equivalent to (y, w) solving the centralized problem

$$\begin{aligned} \underset{y,w}{\texttt{maximize}} \quad & \sum_{i \in \mathcal{I}} \pi_i \\ \texttt{subject to} \quad & y_i \geqslant 0 \quad \forall i \in I \\ & w_i \geqslant 0 \quad \forall i \in I. \end{aligned} \tag{16}$$

In other words, the n individual optimization problems in 2 variables are equivalent to one centralized optimization problem in $2n$ variables. Fourth, (y, w, q, s) satisfy conditions (1, 2) above —characterizing temporary equilibria— if and only if (y, w) solve (16) with the added constraint $\sum w_i = \mathcal{TAC}$ and q is a Lagrange multiplier for that constraint.

A classic result from the cap-and-trade theory with perfect monitoring is that we can attain optimal outcomes that could be obtained in a centralized way by setting up a cap on total output and a market for output quotas. We will investigate whether a similar result carries over to the case of costly imperfect monitoring in Section 4.

4 The Single-Owner Problem and ITQ Implementability

Consider a central manager that is charged with running a quota-managed fishery for the fishermen. Suppose the manager solves the following problem.

$$\begin{aligned} \underset{y,w,s,M}{\texttt{maximize}} \quad & \sum_{i=1}^{n} [py_i - c(y_i, s, \theta_i)] - M \\ \texttt{subject to} \quad & y, w, s, M \geqslant 0 \\ & \sum_{i=1}^{n} y_i \leqslant g(s) \\ (IC) \quad & y_i \in \operatorname{argmax} py_i - c(y_i, s, \theta_i) - \phi(M, y_i, w_i) \qquad i = 1, \ldots, n \end{aligned} \tag{17}$$

This is essentially a moral-hazard problem with imperfect monitoring where the principal is the central manager and the agents are the fishermen.

For simplicity, we will assume only in this Section that the monitoring function ϕ^+ is smooth at zero violations; that is, when $y_i = w_i$, the derivatives $D_2\phi(M, y_i, w_i)$ and $D_3\phi(M, y_i, w_i)$ are uniquely defined and are equal to zero. That makes the problem smoother, and focuses on the interesting case here, which is the case when there are quota violators in the industry. We also assume that ϕ is strictly convex in y_i, which guarantees the existence of certain useful derivatives, notably $D_{w_i} y_i^*$ and $D_M y_i^*$ in (19).

Under our Assumptions, for every (w_i, s, M) there exists a single y_i^* that satisfies the IC constraint. We can rewrite the manager's problem (17) by changing y_i to this optimal y_i^*, and optimizing only over w, s, M. The Lagrangian would then be

$$L = \sum_i [py_i - c(y_i^*, s, \theta_i)] - M + \sum_i \mu_i^y y_i + \sum_i \mu_i^w w_i + \mu^s \left(g(s) - \sum_i y_i^* \right) + \mu^M M \tag{18}$$

In that case, the system of equations characterizing equilibrium (aside from complementary slackness conditions and original constraints) would be given by the fishermen's IC's FOC on

y and the manager's FOC on w, s, M. In that order:

$$\begin{aligned} p - D_1 c_i - D_2 \phi_i + \mu_i^y &= 0 \qquad i = 1, \dots, n \\ D_{w_i} y_i^* \left(p - D_1 c_i - \mu^s \right) + \mu_i^w &= 0 \qquad i = 1, \dots, n \\ \sum_i \left[D_s y_i^* (p - D_1 c_i - \mu^s) \right] + \mu^s g'(s) &= 0 \\ \sum_i \left[D_M y_i^* (p - D_1 c_i - \mu^s) \right] - 1 + \mu^M &= 0 \end{aligned} \tag{19}$$

Remark 2. Note agent i violates his quota if and only if $D_{w_i} y_i^* > 0$ or, equivalently $D_M y_i^* < 0$. It follows that the summation in the last equation in (19) (the first-order-condition in M) has positive terms only on violators, and therefore, it cannot be satisfied with $M > 0$ if there are no violators. That shows that in any given solution to the single-owner problem, at least one agent must be a quota violator.

However, it cannot be the case that *all* violators have positive quota; indeed, for any such i we would have $y_i > w_i > 0$, and thus by the second equation (first-order-condition on w_i), it must be the case that $p - D_1 c_i - \mu^s = 0$. Therefore, if all violators had positive quota, the first-order condition on $M > 0$ also would not be satisfied.

We conclude that in the single-owner solution, either $M = 0$ or there must be a quota violator that is given zero quota.

The intuition is clear for the case $n = 1$: the objection function in (17) tells us that, all else equal, we want to spend as little money as possible with monitoring. Therefore, it is cheaper to reduce the fisherman's output with a lower $\mathcal{TAC}$ than with a higher M. It follows that we should set $w_1 = 0$.

4.1 ITQ Implementability of the Single-Owner problem

We will now show that a quota market can implement the optimal single-owner solution as long as the fines depend on catch y_i and quota w_i only through the absolute violation $y_i - w_i$.

Theorem 1. *Let $(\widetilde{y}, \widetilde{w}, \widetilde{s}, \widetilde{M} > 0)$ be a solution of the single-owner problem. Then there exists $q \geqslant 0$ such that $(\widetilde{y}, \widetilde{w}, q, \widetilde{s})$ is an ITQ equilibrium at monitoring level $\widetilde{M}$ and cap $\sum \widetilde{w}_i$ if $D_2 \phi(M, y, w) = -D_3 \phi(M, y, w)$ for all $M > 0$, $y > w \geqslant 0$.*

In other words, the single-owner optimal solution is ITQ implementable if the penalty function ϕ depends on y, w only through the absolute violation $y - w$.

Proof. Suppose that $D_2 \phi(M, y, w) = -D_3 \phi(M, y, w)$ for all $M > 0$, $y > w \geqslant 0$. Let $\widetilde{\mu}^y, \widetilde{\mu}^w \geqslant 0$ be the Lagrange multipliers of the nonnegativity constraints in the single-owner problem (17) and

$\widetilde{\mu}^s \geqslant 0$ the multiplier of the output/stock-growth constraint. Define

$$\begin{aligned} y_i &= \widetilde{y}_i \\ \mu_i^y &= \widetilde{\mu}_i^y \\ w_i &= \widetilde{w}_i \\ \mu_i^w &= \widetilde{\mu}^s + D_3\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i) \\ q &= \widetilde{\mu}^s \\ s &= \widetilde{s} \end{aligned}$$

A quick check reveals that (y, w, q, s) satisfy the system of equations that characterize an ITQ equilibrium (see p. 8) at monitoring level $\widetilde{M}$ and cap $\sum \widetilde{w}_i$ as long as $w_i, \mu_i^w \geqslant 0$ satisfy the complementary slackness condition $\mu_i^w w_i = 0$. To verify that, remember that $D_3\phi_i = D_2\phi_i$, and thus

$$\mu_i^w = \widetilde{\mu}^s - D_2\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i)$$

The first-order conditions on w_i for the single-owner problem ((19), second equation) guarantee that $\mu^s - D_2\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i) \geqslant 0$, because $D_{w_i} y_i^*$ is always no non negative and $\widetilde{\mu}_i^w \geqslant 0$. Therefore $\mu_i^w \geqslant 0$. Furthermore, if $w_i > 0$, then $\widetilde{\mu}_i^w = \widetilde{\mu}_i^y = 0$ and $D_{w_i} y_i^* > 0$ at the single-owner solution, which implies by (19) (first two equations) that $\widetilde{\mu}^s - D_2\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i) = 0$. Therefore, $\mu_i^w w_i = 0$, as desired. □

From the point of view of the manager, every situation she faces is characterized by p, c, g, ϕ, θ satisfying our Assumptions; those form the set of possible environments.

The manager chooses $M, \mathcal{TAC}, s, y, w$ to induce an industry-profit-maximizing outcome (y, s, M). While she chooses M directly, $s \in [0, K]$ and $y \in \mathbb{R}_+^n$ must satisfy steady state and incentive compatibility conditions. We say an outcome (y, s, M) is ***ITQ implementable*** if there exists $\widetilde{M}, \mathcal{TAC}$ and an ITQ equilibrium $(\widetilde{y}, \widetilde{w}, \widetilde{q}, \widetilde{s})$ such that $\widetilde{y} = y, \widetilde{s} = s, \widetilde{M} = M$.

We showed in Theorem 1 that for any environment with a monitoring function that depends on y_i, w_i only through $y_i - w_i$, every solution to the manager problem is ITQ implementable. We now show that this is a rather special feature of that particular monitoring function. In other words, the impossibility of ITQ-implementation is a generic property (see definition in Appendix C). The intuition behind the proof is simple: a solution of (17) requires that $D_2\phi_i$ be equalized for all violators, while an ITQ equilibrium only guarantees that $D_3\phi_i$ are equalized among quota holders. Intuitively, that should not hold for most monitoring functions ϕ^+. The economic interpretation is the following: the single-owner solution requires that marginal costs be equalized for all violating quota holders; we can see from the first-order condition on y_i that this will happen only if the marginal fine $D_2\phi_i$ is equalized among such agents. However, an *ITQ* equilibrium only equalizes the marginal benefit of quota $-D_3\phi_i$ among those agents. Therefore, if $-D_3\phi = D_2\phi$, then in an ITQ equilibrium, we also equalize marginal costs $D_1 c_i$ among violating quota holders.

Theorem 2. *Fix p, c, g, θ and let Φ be a set of monitoring functions ϕ^+ that are smooth at zero violations, are restricted to some compact domain, and such that any solution $(M, \mathcal{TAC}, s, y, w)$ of the single-owner problem (17) hands out strictly positive aggregate quota $\sum w_i$. Given a solution (y, s, M) to the single-owner problem (17), let $\Phi' \subset \Phi$ be the subset of monitoring functions for which we can implement (y, s, M) with an ITQ program. Then $\Phi \setminus \Phi'$ is a generic set in Φ.*

Proof. First remember that in this Section we are assuming that ϕ^+ is differentiable at zero violations, and therefore every active fisherman in an ITQ equilibrium is a violator. Clearly, if a solution to the single-owner problem has active fishermen respecting their quota, then that solution is not ITQ implementable. For that reason, we restrict attention to solutions of the single-owner problem where all active fishermen are violators.

If a solution to the single-owner problem is ITQ implementable, then the square system of equations (8–6) has to be satisfied. In addition, it follows from the second equation in (19) that $p - D_1 c_i$ has to be equalized for all violators with positive quota. We will now argue that this is generally impossible. The intuition for that generic impossibility is that we need to satisfy a smooth system with more equations than variables. Let us make that precise.

Fix $k \geqslant 2$ and let $\widetilde{\Phi}$ be the vector space of all C^k functions $(M, y, w) \mapsto f(M, y, w)$ from a compact set[11] to $\mathbb{R}$ such that $D_2 f = D_3 f = 0$ whenever $y = w$. Equipped with the usual norm of uniform convergence of the function and its k derivatives $\widetilde{\Phi}$ is a Banach space. It is also separable because the set of all polynomials on that same domain is dense in $\widetilde{\Phi}$ (as stated by the Stone-Weierstrass Theorem). Now define $\widetilde{\Phi}' \subset \widetilde{\Phi}$ as the set of members of $\widetilde{\Phi}$ (not necessarily members of Φ) where the overdetermined system we just mentioned has a solution. It follows then from the transversality Theorem 4 that $\widetilde{\Phi} \setminus \widetilde{\Phi}'$ is a generic set in $\widetilde{\Phi}$.

It remains to show that $\Phi \setminus \Phi'$ is a generic set in Φ. To that end, consider the set of all C^k strongly convex monitoring functions with a positive parameter, that is, those elements of Φ hose hessian have all eigenvalues greater than some $m > 0$ across their domain. It is not hard to show that this is a dense subset of Φ that is also open in $\widetilde{\Phi}$; it is therefore a generic set in Φ. Because the intersection of two generic sets is also generic, it follows that $\Phi \setminus \widetilde{\Phi}'$ is generic in Φ. From $\Phi' = \Phi \cap \widetilde{\Phi}'$ we conclude that $\Phi \setminus \Phi'$ generic in Φ as desired. □

[11] Realistically, y, w, M are all nonnegative and bounded above, so this technical requirement does not get in the way of realism.

5 The Set of Equilibria

We will now build our equilibrium notion by separating the economic equilibrium and the environmental equilibrium parts. This separation makes the analysis conceptually clearer, and allows us to build the equilibrium in a "bottom-up" way, making clear the interdependence relations between the different variables of the model.

On the economic side, we have two "layers":

- In the bottom layer, we study individual optimal behavior in both fishing activity and trading activity in the quota market. Agents take $p, M, \mathcal{TAC}, q, s$ as given and make their optimal decisions about how much quota w_i to hold and how much fish y_i to catch. This corresponds to the first condition for equilibrium: fishermen maximize profits by solving (4). At this level, the concept of *quota demand* and the distinction between *quota violators* and *quota holders* are central.

- On top of the previous layer, we impose the *market-clearing* condition (5). At this level, outcomes are denominated *temporary equilibria*, and a lot of what we want to know is summarized in the variables that quantify total output Y and quota price q. The parameters $p, M, \mathcal{TAC}, s$ are still exogenous.

On the environmental side, we have the final layer.

- If we impose the steady-state condition (6), we obtain the definitive notion of *equilibrium* in this model. At this level, the only free parameters are the price of fish p and the regulator's design variables M and $\mathcal{TAC}$.

The center of our approach lies in the Section on temporary equilibria; once that is well understood, the results about existence, number and stability of equilibria can be inferred after a graphical analysis of the growth curve $s \mapsto g(s)$ and the temporary equilibrium total output curve $Y(s)$.

5.1 Competitive Behavior: Fishing and Quota Demand

From a fisherman i's point of view, he takes $M, \mathcal{TAC}, \omega_i, s, q$ as given and his choice of y_i, w_i leads to a certain private outcome (y_i, w_i, q, s) with an associated profit π_i.

Let us define some terminology.

- We define for every $i \in \mathcal{I}, q \geqslant 0$ and $s \geqslant 0$ the ***optimal catch*** $\widetilde{y}_i(q,s)$ and ***optimal quota holdings*** $\widetilde{w}_i(q,s)$ as the $y_i \geqslant 0$ and $w_i \geqslant 0$ that maximize profits π_i as defined in (2).

- Given a stock s, note that ***open-access*** behavior is given by fishermen's optimal behavior at $q = 0$. That is $\widetilde{y}_i(0, s)$ is agent i's open-access catch when the stock is s. Analogously, the ***pure-poaching*** catch of agent i at stock s is equal to $\widetilde{y}_i(\infty, s)$.

We say that a fisherman i is ***active*** when $y_i > 0$; we say he is a ***violator*** when $y_i > w_i$.

In Appendix B, p. 30, we characterize the agents that will be active and those who will hold positive amounts of quota. Based on Lemma 11 presented there, we can also say something about who will violate quota, and who will not. It is easy to see, for example, if $q > 0$ and $D_3\phi = 0$ when violations $y_i - w_i$ are zero, then any active agents will be violators. This observation applies to the case where the monitoring function ϕ^+ is smooth at zero violations. Theorem 3 below specifies what can happen when this condition fails.

Theorem 3. *Consider arbitrary $M > 0, \mathcal{TAC}, q, s$, and let (y, w) be profit maximizing for all fishermen. Then either no active fisherman violates his quota or every active fisherman violates his quota.*

Proof. We will prove the following equivalent statement: if there is an agent that holds positive quota and does not violate it, then no fisherman violates his quota.

Before we proceed, note that the $\mathcal{TAC}$ is irrelevant here, and any case with $M = 0$, $s = 0$ or $q = 0$ is trivial.

Let i be the non violating quota holder. According to Lemma 11 in Appendix B, his optimal w_i satisfies

$$-q - \alpha_i D_3\phi_i = 0. \tag{20}$$

for some $\alpha_i \in [0, 1]$.

Let j be another fisherman. Suppose, by means of contradiction, that j violates his quota. His optimal w_j satisfies

$$-q - D_3\phi_j + \mu_j^w = 0 \tag{21}$$

for some $\mu_j^w \geqslant 0$. Subtracting (21) from (20), we obtain

$$\mu_j^w = D_3\phi_j - \alpha_i D_3\phi_i$$

Because j violates his quota while i does not, and because we assume that ϕ is strictly convex in quota holdings when violations are positive, (see Assumption 3 in page 6) we must have $D_3\phi_i > D_3\phi_j$ (remember $D_3\phi$ is always non positive). It follows then from $\alpha_i \leqslant 1$ that $\mu_j^w < 0$, which is a contradiction. □

Remark 3. Note that in the proof of Theorem 3 above, it was crucial to assume that the monitoring function ϕ was *strictly* convex in w_j. If ϕ was simply convex in w_j, we would only be able to conclude that $\alpha_i = 1$ and $\mu_j^w = 0$ which does not contradict our knowledge that $\mu_j^w \geqslant 0$.

The following Lemmas are a preparation for Section 5.2 where we study temporary equilibria. They help us establish monotonicity properties of the excess demand function so that quota prices can be uniquely defined in temporary equilibria.

Lemma 1. *Fix $M, \mathcal{TAC}, s, i$ and $q > 0$. Then for every $y_i \geqslant 0$ there is a unique w_i^* that maximizes profits. The implicit mapping $y_i \mapsto w_i^*(y_i)$ is nondecreasing and continuous, differentiable where $w_i^* \neq y_i$ with $w_i^*(y_i) \leqslant y_i$. Furthermore, there is $s_i \geqslant 0$ such that $w_i^*(s) = 0$ if and only if $s \leqslant s_i$. Where $s > s_i$, the map $s \mapsto w_i^*(s)$ is strictly increasing.*

Proof. Existence of w_i^* is a consequence of $\lim_{w_i \to \infty} \pi = -\infty$ and Weierstrass' Theorem; uniqueness follows from strict concavity of the profit function in w_i. Continuity of the implicit mapping is an implication of the maximum Theorem. The monotonicity properties follow from the increasing differences of the profit function in (w_i, y_i). Differentiability is a consequence of the invertibility of the hessian of the profit function —see Lemma 7 in Appendix A. $\square$

Lemma 2. *Fix $M, \mathcal{TAC}, i$. Then for every $q > 0, s \geqslant 0$ there exists a unique pair $\widetilde{y}_i, \widetilde{w}_i$ that maximizes profits; it must be the case that $\widetilde{w}_i \leqslant \widetilde{y}_i$. The maps $(q,s) \mapsto \widetilde{y}_i(q,s)$ and $(q,s) \mapsto \widetilde{w}_i(q,s)$ are continuous, continuously differentiable where $q, s > 0$ and $\widetilde{y}_i > \widetilde{w}_i$ and strictly decreasing in q once $\widetilde{w}_i, \widetilde{y}_i > 0$. The map $\widetilde{y}_i$ is strictly increasing in s once $\widetilde{y}_i > 0$, and $\widetilde{w}_i$ is strictly increasing in s once $\widetilde{w}_i > 0$.*

Proof. Note that the profit function $\pi_i(y_i, w_i) = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i)\phi(M, y_i, w_i)$ is continuous and defined over a closed domain. It is also *coercive*, that is, for all $i \in \mathcal{I}$ and all $t < 0$ there exists $r > 0$ such that if $\|(y_i, w_i)\| > r$ then $\pi_i < t$. We can then bound the domain and appeal to Weierstrass' Theorem to conclude that the set of (y_i, w_i) that maximize π_i is not empty.

Let us now show that the set of profit-maximizing (y_i, w_i) is a singleton. First, take the case where there exists a maximizer such that $w_i > 0$. We know from Lemma 1 that associated to that w_i there is a single profit-maximizing y_i, and that for any other possible maximizer $(\widetilde{y}_i, \widetilde{w}_i)$ we must have $(y_i - \widetilde{y}_i)(w_i - \widetilde{w}_i) > 0$. Suppose $\widetilde{y}_i > y_i$. As we assumed that $D_{11}c > 0$, we have $D_1 c(\widetilde{y}_i, s, \theta_i) > D_1 c(y_i, s, \theta_i)$. The first-order conditions on output are $p = D_1 c + D_2 \phi$. Therefore, we must have $D_2\phi(M, \widetilde{y}_i, \widetilde{w}_i) < D_2\phi(M, y_i, w_i)$. But that implies that $D_3\phi$ also moved, violating the first-order condition on quota holdings $D_3\phi = q$. The argument for the case $\widetilde{y}_i < y_i$ is analogous. That proves that the set of maximizers is a singleton.

The monotonicity results come from the fact that the profit function π_i has increasing differences in (y_i, s) (strict when $y_i, s > 0$), in $(w_i, -q)$ (strict when $w_i, q > 0$, and in (y_i, w_i).

That $\widetilde{y}_i$ and $\widetilde{w}_i$ are continuous functions of q, s follows from the maximum Theorem.

Because the hessian (25) of the profit function is always invertible when $y_i, w_i > 0$ and varies smoothly with $q, s > 0$ it follows from the implicit function Theorem that $\widetilde{y}_i, \widetilde{w}_i$ are smoooth functions of $q > 0$ and $s > 0$ when $\widetilde{y}_i > 0, \widetilde{w}_i > 0$. $\square$

Given $M, \mathcal{TAC}$, define the ***excess demand*** function as

$$z(q,s) = \left(\sum_{i \in \mathcal{I}} \widetilde{w}_i(q,s) \right) - \mathcal{TAC} \tag{22}$$

but only for strictly positive values of q. We have then the following Corollary from Lemma 2.

Corollary 1. *The excess demand function in (22) is non increasing in $q > 0$. Furthermore, if $z(q,s) = 0$ with $q > 0$, then there exists an open interval around q where $z(\cdot, s)$ is strictly decreasing.*

Remark 4. We studied $\widetilde{y}_i(q,s)$ and $\widetilde{w}_i(q,s)$ when $q > 0$. Matters are simpler when $q = 0$. In this case a pair (y_i, w_i) maximizes profits if and only if y_i maximizes operational profits $py_i - c(y_i, s, \theta_i)$, and w_i is high enough that $\phi(M, y_i, w_i) = 0$. As we assumed c strictly convex in y_i, it follows that there is only one profit-maximizing y_i.

5.2 Temporary Equilibria

We show in Theorem 4 that a temporary equilibrium always exists at every stock level and that it is unique when the $\mathcal{TAC}$ is lower than the open access catch at that stock level. In Theorem 5, we show that the equilibrium maps $s \mapsto q(s)$ and $s \mapsto Y(s)$ are continuous and nondecreasing.

Theorem 4. *Fix $M, \mathcal{TAC}, s > 0$ arbitrarily. There exists $w \in \mathbb{R}^n_+$ and unique $y \in \mathbb{R}^n_+, q \geqslant 0$ such that (y, w, q, s) is a temporary equilibrium. If $q > 0$, then w is also unique. Furthermore, $q > 0$ if and only if $\sum_i \widetilde{y}_i(0,s) > \mathcal{TAC}$.*

In more detail: if $\sum_i \widetilde{y}_i(0,s) > \mathcal{TAC}$ then there exists unique $q > 0, y, w \in \mathbb{R}^n_+$ such that (y, w, q, s) is a temporary equilibrium. If $\sum_i \widetilde{y}_i(0,s) < \mathcal{TAC}$, then (y, w, q, s) is an equilibrium if and only if $q = 0$, $y = \widetilde{y}(0,s)$, $\sum_i w_i = \mathcal{TAC}$, and $w_i \geqslant y_i$ for all i. Finally, if $\sum_i \widetilde{y}_i(0,s) = \mathcal{TAC}$, then (y, w, q, s) is an equilibrium if and only if $y = w = \widetilde{y}(0,s)$ and $q = 0$.

Proof.

Case 1. First, suppose $\mathcal{TAC} - \sum \widetilde{y}_i(0,s) < 0$. We claim excess demand of quota will be positive if $q = 0$. Indeed, if y_i, w_i solve (4), then $q = 0$ implies $w_i \geqslant y_i$, and thus $z(q = 0, s) > 0$. Therefore, we can restrict our search to temporary equilibria with positive quota price. We know that $z(q,s) \to -\mathcal{TAC}$ as $q \to \infty$, and thus by the intermediate value Theorem there will be $q > 0$ such that $z(q,s) = 0$. It follows from Corollary 1 that there is only one such q. Furthermore, in an equilibrium with $q > 0$, we must have $w_i \leqslant y_i$ for all i. Therefore, profits π_i are strictly concave in (y_i, w_i) thus proving the uniqueness of the temporary equilibrium at s.

Case 2. In case $\mathcal{TAC} - \sum \widetilde{y}_i(0,s) > 0$, there can be no temporary equilibrium with $q > 0$. To see that, notice that at positive price $w_i \leqslant y_i$ at a solution of the fishermen problems (4). Therefore, excess quota demand will be negative if $q > 0$. We will now show that there is a temporary equilibrium with $q = 0$. Set $q = 0$ and let

$$S = \{(y, w) \in \mathbb{R}^n_+ \times \mathbb{R}^n_+ : (y, w) \text{ solve } (16)\}$$

As for all $(y, w) \in S$ and any $w' \geqslant y$ we have $(y, w') \in S$ we conclude that there are points (y, w)in S such that $\sum w_i = \mathcal{TAC}$ because $\sum \tilde{y}_i(q = 0, s) \leqslant \mathcal{TAC}$. Let (y, w) be any such points. By construction, the tuple (y, w, q, s) satisfies all the conditions for a temporary equilibrium. Furthermore, we cannot have any $w_i < y_i$ in a temporary equilibrium with $q = 0$ because fisherman i could increase his profit by buying more quota for free.

Case 3. Finally, $\mathcal{TAC} - \sum \tilde{y}_i(0, s)$ may be zero. In that case, a natural equilibrium candidate is (y, w, q, s) with $q = 0$, $y = w = \tilde{y}(0, s)$. We can verify that (y, w, q, s) is indeed a temporary equilibrium by checking that there exists $\mu_i^y, \mu_i^w \geqslant 0$ for all $i \in \mathcal{I}$ such that conditions (14–15) are satisfied with $\alpha_i = 0$ for all i. In fact, we can show that this is the only temporary equilibrium. To see that, suppose (y, w, q, s) is a temporary equilibrium. Because $\mathcal{TAC} > 0$, some fisherman i must have positive quota $w_i > 0$. As $y_i = w_i$, his choice of y_i must be an interior maximizer of operational profits $py_i - c(y_i, s, \theta_i)$ It follows from first-order conditions that, $p = D_1 c(y_i, s, \theta_i)$. Therefore, condition (14) holds only if $\alpha_i = 0$. It follows from (15) that $q = 0$, and therefore $y = w = \tilde{y}(0, s)$.

□

Lemma 3. *Given $M, \mathcal{TAC}$ and under our simplifying Assumptions, the temporary equilibrium maps $s \mapsto y(s)$, $s \mapsto w(s)$ are continuous. The temporary equilibrium map $s \mapsto q(s)$ is continuous on all points but those s^* where $Y(s^*) = \mathcal{TAC}$; where that happens, $q(s^*)$ might be a set, but if marginal violations $D_2\phi$ are zero when violations $y_i - w_i$ are zero, then $q(s^*)$ is a point, and q is continuous at s^*.*

Proof. Fix M and $\mathcal{TAC}$ arbitrarily. The tuple (y, w, q, s) is a temporary equilibrium if and only if y, w solve

$$\begin{aligned} \underset{y,w}{\texttt{maximize}} \quad & \sum_{i \in \mathcal{I}} py_i - c(y_i, s, \theta_i) - \phi(M, y_i, w_i) \\ \texttt{subject to} \quad & y_i \geqslant 0 \quad \forall i \in I \\ & w_i \geqslant 0 \quad \forall i \in I \\ & \mathcal{TAC} - \sum_{i \in \mathcal{I}} w_i = 0 \end{aligned} \tag{23}$$

and q is equal to the multiplier of the last constraint. It follows from the maximum Theorem that the temporary equilibrium maps $s \mapsto y(s)$ and $s \mapsto w(s)$ are continuous functions.

Let us now prove that $s \mapsto q(s)$ is continuous. Define the set

$$V(\tilde{s}) = \{i \in \mathcal{I} : y_i(\tilde{s}) > w_i(\tilde{s})\}$$

as the set of *violators* and

$$H(\tilde{s}) = \{i \in \mathcal{I} : w_i(\tilde{s}) > 0\}$$

as the set of *quota holders*. Suppose that $Y(\tilde{s}) > \mathcal{TAC}$. This Assumption guarantees that $V(\tilde{s}) \neq \emptyset$ and the constraint $\mathcal{TAC} - \sum w_i = 0$ in the problem above guarantees that $H(\tilde{s}) \neq \emptyset$. Theorem 3 guarantees that $\emptyset \neq H(\tilde{s}) \subset V(\tilde{s})$. Therefore we can pick a fisherman j in $V(\tilde{s}) \cap H(\tilde{s})$.

Because of the continuity of the maps $y(s), w(s)$ and the fact that $q(s) > 0$ whenever $Y(s) > \mathcal{TAC}$ we can find an open interval U containing $\tilde{s}$ such that for all $s \in U$

$$j \in V(s) \cap H(s) \text{ and } q(s) > 0$$

Therefore, using the optimality condition on w_i in the problem above, we conclude that for all $s \in U$

$$q(s) = -D_3\phi(M, y_i(s), w_i(s))$$

It follows from the continuity of $D_3\phi$ that $s \mapsto q(s)$ is a continuous function on U, as desired. From here, the cases where $Y(s) \leqslant \mathcal{TAC}$ are straightforward. □

Theorem 5. *Given $M, \mathcal{TAC} > 0$ the temporary equilibrium maps $s \mapsto q(s)$ (aside from, possibly, a point where $(Y(s) = \mathcal{TAC})$) $s \mapsto Y(s)$ are non decreasing, $Y(s) > \mathcal{TAC}$ implies $q(s) > 0$, and $Y(s) < \mathcal{TAC}$ implies $q(s) = 0$. Furthermore, where $q, Y > 0$, those maps are strictly increasing in s. If $D_2\phi = 0$ when violations $y_i - w_i$ are zero, then $Y(s) = \mathcal{TAC}$ implies $q(s) = 0$.*

Proof. Remember that $q(s)$ and $Y(s)$ are the quota price and total output associated with the temporary equilibrium at s. Note that as s increases, marginal costs $D_1c(y_i, s, \theta_i)$ (for fixed y_i) go down, and thus the marginal benefit of violating the constraint $\mathcal{TAC} - \sum w_i \geqslant 0$ cannot go down. Therefore, $q(s)$ cannot be decreasing in s. Indeed, as we assumed that $D_{21}c(y_i, s, \theta_i) < 0$ for all s, it follows that the marginal benefit of violating the constraint *goes up* once there is any benefit at all. By the same token, $Y(s)$ cannot be decreasing in s and is strictly increasing once $Y(s) > 0$.

Note that for all s, the temporary equilibrium $y_i(s)$ is given by the optimal catch $\tilde{y}_i(q(s), s)$. Now, fix s such that $Y(s) > \mathcal{TAC}$. As $Y(s) = \sum_i \tilde{y}_i(q(s), s) \geqslant \sum_i \tilde{y}_i(0, s)$, we must have $\sum_i \tilde{y}_i(0, s) > \mathcal{TAC}$. It follows from Theorem 4 that $q(s) > 0$. We now prove the converse via its contrapositive. Fix s satisfying $Y(s) < \mathcal{TAC}$. Then a solution of problem (23) is also a solution of the same problem without the constraint $\sum_i w_i = \mathcal{TAC}$. Therefore, 0 has to be a multiplier of that constraint, and thus a temporary equilibrium quota price q at s. As we already showed in Theorem 4 that for every $s \geqslant 0$ and $\mathcal{TAC} > 0$ there is at most one temporary equilibrium $q(s)$, we conclude that $q(s)$ has to be zero. □

Corollary 2. *Fix $M, \mathcal{TAC} > 0$. There exists at most one point $\bar{s} \geqslant 0$ such that $Y(\bar{s}) = \mathcal{TAC}$. If such a point exists, then*

$$s < \bar{s} \Longrightarrow q(s) = 0$$
$$s > \bar{s} \Longrightarrow q(s) > 0$$

Lemma 4. *The temporary equilibrium variables y, w, q are smooth functions of $p, M, \mathcal{TAC}, q, s$ as long as $y_i > w_i > 0$ for all agents i.*

Proof. Follows from the implicit function Theorem. See Lemma 9 in Appendix A. □

Lemma 5. *Fix $\bar{s}, \overline{M}, \overline{\mathcal{TAC}} > 0$ and an associated temporary equilibrium $(\bar{y}, \bar{w}, \bar{q}, \bar{s})$ such that every active fisherman violates his quota. There exits an open neighborhood $V \times W$ of $(\overline{M}, \overline{\mathcal{TAC}})$ where the temporary equilibrium map $M \mapsto Y(M, \overline{\mathcal{TAC}}, \bar{s})$ is decreasing and the temporary equilibrium map $TAC \mapsto Y(\overline{M}, \mathcal{TAC}, \bar{s})$ is increasing.*

Proof. We already showed in Lemma 4 that those maps are differentiable. The monotonicities are very intuitive, and can be verified by computing the partial derivatives. □

5.3 Steady-State Equilibria

An outcome (y, w, q, s) is an equilibrium when it is a temporary equilibrium and $Y = g(s)$. Thus, with the results from Section 5.2 we can now visualize the set of equilibria by making a superimposed plot of the maps $s \mapsto g(s)$ and $s \mapsto Y(s)$ where $Y(s)$ is the total output of the temporary equilibrium at s. We say an outcome $(y, w, q, s)$is a ***high-stock equilibrium*** if $s \geqslant sMSY$ where $sMSY = \operatorname{argmax} g(s)$ is the stock associated with the maximum sustainable yield. We say that the outcome is a ***low-stock equilibrium*** if $s < sMSY$.

Remark 5. For fixed $\mathcal{TAC}, M$, an equilibrium with stock at least $s \geqslant sMSY$ exists if and only if $Y(s) < g(s)$. If such an equilibrium exists, then it is the only high-stock equilibrium. It follows that if the pure-poaching catch $\widetilde{Y}(\infty, sMSY)$ is larger than the maximum sustainable yield MSY, then a high-stock equilibrium is attainable only if the monitoring expenditure is raised. In other words, if closing the fishery cannot bring the total catch to levels below MSY, then a high-stock equilibrium will not be attained without more investment in monitoring.

Remark 6. Given a fixed monitoring expenditure M, we can use the Theorems in Section 5.2 to classify possible instances of our model in three categories:

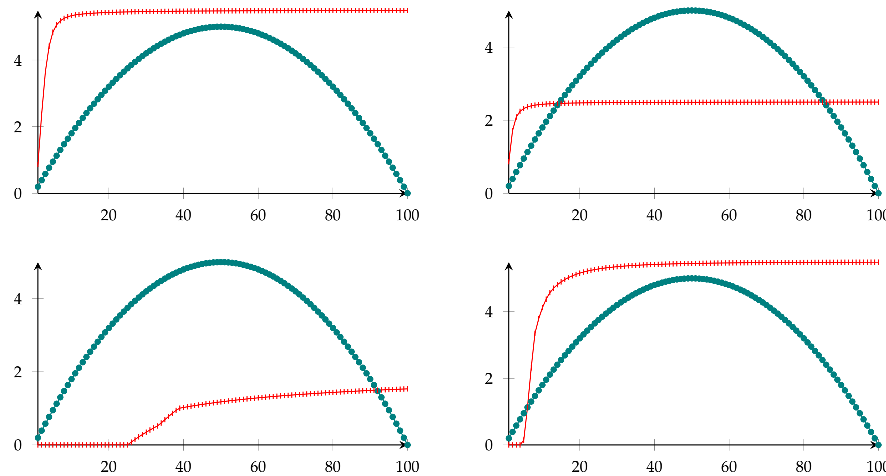


Figure 1: Temporary equilibrium graphs

- *High cost of fishing the $\mathcal{TAC}$.* In this case, agents can never sustain a low-stock equilibrium with positive stock, that is, $Y(s) < g(s)$ for all $s \in (0, sMSY)$. Here, we have one and only one equilibrium besides the one at $s = 0$, and it presents a high stock level.

- *Low cost of fishing the $\mathcal{TAC}$.* In this case, the agents are productive enough to sustain a low-stock equilibrium, that is, there exists $s \in (0, sMSY)$ such that $Y(s) = g(s)$. While one might think that a high-stock equilibrium should exist too, that need not be the case, as shown in figure 1.

- *Extremely low cost of fishing the $\mathcal{TAC}$.* In this case fishermen always want to fish more than the environment can provide, that is, $Y(s) > g(s)$ for all $s > 0$. The only equilibrium is total stock collapse at $s = 0$.

See figure 1 for some illustrations.[12]

We say an equilibrium $(\overline{y}, \overline{w}, \overline{q}, \overline{s})$ is ***degenerate*** when the temporary equilibrium curve $s \mapsto Y(s)$ is tangent to the stock-growth curve $s \mapsto g(s)$ at $\overline{s}$. An equilibrium is ***regular*** when it is not

[12] Both x and y axis are measured in tons of fish. The parabola in green is the stock-growth function $g(s)$, and the other curve in red is the temporary equilibrium total output function $Y(s)$. In the top left, the agents are "too productive" and the only equilibrium is stock collapse. In the top right, there is a low-stock, unstable equilibrium and a high-stock stable equilibrium. In the bottom left, the agents are "not very productive" in the sense that there is a stable high-stock equilibrium and no low-stock equilibrium besides collapse. In the bottom right, there is a minimum stock level for positive production, and that allows the emergence of a *stable* low-stock equilibrium. All plots were generated by numerically computing the temporary equilibrium at $s = 0, 1, 2, \ldots, 100$ for different instances of the model.

degenerate. From now on, we will focus on regular equilibria because they are the only outcomes from this model that we could, in principle, actually observe. We will make that precise now.

A necessary and sufficient condition for regularity of an equilibrium with violators is that the Jacobian of the equilibrium system (8–6) (depicted in page 26) be invertible. The implicit function Theorem allows us to write regular equilibria locally as a continuously differentiable function of the parameters $p, M, \mathcal{TAC}$.

It follows that, starting from a regular equilibrium, if we perturb the parameters $p, M, \mathcal{TAC}$ "just a little bit", we will obtain a new equilibrium that is also regular. This is graphically intuitive. Perturbing the parameters $p, M, \mathcal{TAC}$ entails perturbing the temporary equilibrium curve $s \mapsto Y(s)$; it should not be surprising then that if a curve $s \mapsto Y(s)$ at first crosses the growth curve $s \mapsto g(s)$ then a perturbation of Y should also cross g.

Degenerate equilibria on the other hand do not have that property. Intuitively, if the temporary equilibrium catch curve $s \mapsto Y(s)$ is tangent to the growth curve $s \mapsto g(s)$ at $\bar{s}$, then a small perturbation of the parameters can lead to either the curves crossing each other, or not intersecting at all.

Real-world measurements of Y, s, g and other variables always contain some error. In that sense, it is virtually impossible to observe a degenerate equilibrium. Theorem 6 formalizes the discussion up to here.

Theorem 6. *Almost all equilibria are regular. More formally, consider an open set $\Gamma \subset \mathbb{R}^3$ of tuples $(p, M, \mathcal{TAC})$ for which equilibrium exists. Let $\Gamma' \subset \Gamma$ be the set of such tuples where the associated equilibria are regular. Then $\Gamma \setminus \Gamma'$ is a nowhere dense set of measure zero.*

Proof. Locally, this is a consequence of the implicit function Theorem and Lemma 10 in Appendix A. We can then globalize the result because Γ is separable and the countable union of sets of measure zero has measure zero. Nowhere denseness is a local property, so we do not have to worry about globalizing that. □

The arguments presented here deal with the problem of *determinacy of equilibrium*. See Debreu (1970) for the start of this literature in economic theory and Shannon (2008) for a comprehensive survey that includes pointers on how to extend these arguments to the nonsmooth case.

5.4 Stability of Equilibrium under Myopic Dynamics

Here we analyze what happens to equilibria when we slightly perturb the stock level. The dynamics are determined by

$$s' = g(s) - Y(s) \tag{24}$$

where $Y(s)$ is the temporary equilibrium total output at stock s, so $Y(s) = \sum_i y_i(s)$ where each $y_i(s)$ is in a solution to (4).

We say a regular equilibrium (y, w, q, s) is (locally) ***stable*** when it is robust to small perturbations in the stock level s. A necessary and sufficient condition for that is $g'(s) < Y'(s)$. In other words, at a stable equilibrium, the temporary equilibrium curve Y cuts the stock-growth curve g from below. It follows that high-stock equilibria are always locally stable, while low-stock equilibria may not be.

Remark 7. Stable equilibria are the only outcomes of this model that could, in principle, be "credible" outcomes in a steady state. Indeed, while we do not incorporate stock shocks into our model, in reality they do exist; the only equilibria that can persist under such shocks are stable ones. It is reassuring then to know that high-stock equilibria are always stable. However, as figure 1 shows, very inefficient, low-stock equilibria can also be stable. If this model is a suitable approximation of reality, then this last observation suggests that, if a high-stock equilibrium is desired, then fisheries with severely depleted stocks should cease activities for some time to allow the stock to recover before a cap-and-trade system is put in place.

6 Comparative Statics: Varying the Design Variables

We now investigate how changing the monitoring level $M \geqslant 0$, the $\mathcal{TAC} \geqslant 0$ and quota endowments $\omega \in \mathbb{R}^n_+$ affects equilibrium.

Theorem 7. *Fix $(p, M, \mathcal{TAC}) \in \Gamma'$ as defined in Theorem 6. Fix an associated regular, stable equilibrium (y, w, q, s) with positive violations. The equilibrium variables Y, q and s have the following local monotonicity properties:*

- *The stock s is increasing in the monitoring level M.*
- *All other parameters equal, q varies in the same direction as s.*
- *At a high-stock equilibrium, Y is decreasing in M.*
- *At low-stock equilibria, Y is increasing in M.*
- *Lowering the $\mathcal{TAC}$ changes Y, q, s in the same direction as raising M.*

Proof. From what we discussed in Section 5.3, we know that regular equilibria (y, w, q, s) are locally continuously differentiable functions of M and $\mathcal{TAC}$. Therefore, we can pin down the monotonicity results we want by analyzing the appropriate derivatives.

Making explicit the role of M in the steady-state equation (6), we obtain

$$g(s(M, \mathcal{TAC})) - Y(s(M, \mathcal{TAC}), M, \mathcal{TAC}) = 0$$

differentiating that with respect to M we obtain

$$D_M s = \frac{Y_M}{g' - Y_s}$$

where Y_s, Y_m are *partial* derivatives of the *temporary equilibrium* map $(s, M, \mathcal{TAC}) \mapsto Y(s, M, \mathcal{TAC})$ and g' is the derivative of the growth function $s \mapsto g(s)$. As $Y_M < 0$ (remember, this is a temporary equilibrium map, so it just measures the direct effect of M on total catch) it follows that the sign of $D_M s$ is the sign of $Y_s - g'$. Remember that Y_s is the slope of the temporary equilibrium map $s \mapsto Y(s)$, and thus at stable equilibria, $Y_s - g'$ must be strictly positive. It follows that $D_M s > 0$, as desired.

Remember from Theorem 5 that the direct effect of s on q is positive and because q is the Lagrange multiplier of the market-clearing constraint in (23) it must be the case that the direct effect of M on q is positive. Differentiating the equilibrium map $(M, \mathcal{TAC}) \mapsto q(s(M, \mathcal{TAC}), M, \mathcal{TAC})$ with respect to M we conclude that $D_M q = q_s D_M s + q_M$. We conclude from what we just discussed and the previous paragraph that $D_M q > 0$.

The sign of $D_{\mathcal{TAC}} s$ and $D_{\mathcal{TAC}} q$ can be obtained analogously, and it should be clear by now why their signs are opposite to the signs of the corresponding M-derivatives.

The monotonicity on Y is graphically intuitive: raising M lowers the temporary equilibrium curve $s \mapsto Y(s)$; therefore, it will intercept the graph of g at lower points.

The effects of the $\mathcal{TAC}$ are opposite because raising it pushes the temporary equilibrium curve $s \mapsto Y(s)$ up. □

7 Larger Agents vs. Smaller Agents

In order to compare equilibrium outcomes across agents, we will need to make further Assumptions.

Assumption 4. The type space Θ is an interval of real numbers. Given agents $i, j \in \mathcal{I}$, we say i is ***larger than*** j or that j is ***smaller than*** i when i has a higher fixed cost and a lower marginal cost than j; we indicate that by setting their types $\theta_i > \theta_j$. Given our Assumptions, the geometrical meaning is that the marginal cost curve $y_i \mapsto D_1 c(y_i, s, \theta_i)$ of a given agent i is always below the marginal cost curve of agents smaller than i.

Assumption 5. The larger agents bear the brunt of the externality: for all $y, s \geqslant 0$ we have $D_2 c(y, s, \theta)$ decreasing in θ. Remember $D_2 c$ is always negative. The intuition is that, holding the catch fixed, with a big boat costs rise sharply if the stock decreases, or equivalently, costs fall quickly if the stock increases. One can imagine that with a very low stock, both a big boat and a small boat would need to spend more or less the same number of hours in the water, while with high stock the big boat can catch a lot of fish very quickly on the same spot, spending less time in the water than the small boat. The cost increases or decreases are then explainedby the fact that a big boat has higher operational costs per hour.

Lemma 6. *In equilibrium, larger agents produce more and buy more quota. Precisely, if $\theta_i > \theta_j$, then in equilibrium $w_i \geqslant w_j$ and $y_i \geqslant y_j$. Furthermore, if $w_i > 0$, then these inequalities are strict.*

Proof. Let $\theta_i > \theta_j$ as in the Theorem statement. Note that π has strictly increasing differences in (y_i, θ_i) and (weakly) increasing differences in (w_i, θ_i) and (y_i, w_i). Therefore, if y_i, y_j maximizes the profits of fishermen i, j, we must have $y_i \geqslant y_j$. Finally, by Lemma 1, it follows that $w_i \geqslant w_j$. □

7.1 Varying the Design Variables

The message of Theorem 8 below is the following: if wealth effects are absent (identical net trades of quota across agents) or larger for larger agents, then larger agents benefit marginally from monitoring more than smaller agents. For this result, we will need to assume that ϕ depends on y_i, w_i only through the absolute violation $y_i - w_i$).

Theorem 8. *Fix an interior equilibrium. Let $\theta_i > \theta_j$ and suppose wealth effects are absent or larger for the larger agent, that is, $\omega_i - w_i \geqslant \omega_j - w_j$. Then $D_M \pi_i > D_M \pi_j$.*

Proof. Suppose $\omega_i - w_i = \omega_j - w_j$ for simplicity. It follows from Lemma 6 that $y_i > y_j$. As $D_M \pi_i = D_M q(\omega_i - w_i) - D_M s D_2 c(y_i, s, \theta_i) - D_1 \phi(M, y_i, w_i)$,

$$\begin{aligned} D_M \pi_i - D_M \pi_j &= D_M s \left(D_2 c(y_j, s, \theta_j) - D_2 c(y_i, s, \theta_i) \right) \\ &= D_M s \left(D_2 c(y_j, s, \theta_j) - D_2 c(y_j, s, \theta_i) + D_2 c(y_j, s, \theta_i) - D_2 c(y_i, s, \theta_i) \right) \end{aligned}$$

It follows from our Assumptions that both the first difference and the second difference in the parentheses above are positive. Therefore, as $D_M s > 0$ by Theorem 7, $D_M \pi_i - D_M \pi_j > 0$, as desired. It should be clear now that if $\omega_i - w_i > \omega_j - w_j$ then the gap $D_M \pi_i - D_M \pi_j > 0$ is further widened. □

Theorem 9. *Fix an interior equilibrium. Let $\theta_i > \theta_j$ and suppose wealth effects are absent or larger for the larger agent, that is, $\omega_i - w_i \geqslant \omega_j - w_j$. Then $D_{\mathcal{TAC}} \pi_i < D_{\mathcal{TAC}} \pi_j$.*

Proof. Analogous to the proof of Theorem 8, but note that $D_{\mathcal{TAC}}s < 0$ according to Theorem 7. □

8 Final Remarks

In this paper we studied individual tradable quota (ITQ) programs for the exploitation of renewable resources when monitoring is imperfect and costly. We examined how the welfare, political and stability properties of equilibria change as we vary the intensity and technology of enforcement or the level of the cap. We also investigated the properties of the second-best solution to the fishery problem and of the multiple steady-state equilibria that may arise in an ITQ program.

We learned that the optimal single-owner choice of enforcement and cap is associated with positive quota violations and that in an ITQ equilibrium either nobody or everyone violates quota. We also learned that if expected fines ϕ depend on the catch y_i and quota held w_i only through the absolute violation $y_i - w_i$, then the second-best outcome is ITQ immplementable, and, if quota holdings are "right", then larger boats are more likely to want more enforcement and a lower cap.

Finally, we saw how the initial allocation of quota —while incapable of affecting equilibria directly— can *indirectly* affect outcomes if fishermen can influence the choice of the cap and enforcement levels.

We believe these results provide a foundation for what is the next step in this research agenda: a good mechanism that regulators could use to set the $\mathcal{TAC}$, its distribution among fishermen, and enforcement levels.

It is natural to expect that in most institutional setups, the expenditure on monitoring is largely influenced (if not paid for) by the members of the industry, while the $\mathcal{TAC}$ is insulated from such influence. It is therefore crucial to know, given a certain status quo, which fishermen will support more monitoring, and which will support less monitoring; that knowledge will tell us which outcomes are feasible from a political point of view. While we provide the first steps in that direction, significant work remains ahead: crucially, we do not know when the optimal monitoring level (in the sense of (17)) is supported by a majority of fishermen, a topic left for future research. We believe any new developments in that direction could be of great value to regulators and managers of tradable quota programs.

Appendices

A Invertibility of the Jacobian of Various Equilibrium Subsystems

The smoothness of the temporary equilibrium maps (and later, the equilibrium maps) depend on the invertibility of the Jacobian of the subsystem relevant to temporary equilibrium (for equilibrium, we will need to look at the whole system) The Jacobian of the equilibrium system (8–13) in the case $n = 2$ and $y_1 \neq w_1, y_2 \neq w_2$ is displayed in figure 2.

Figure 2: Jacobian of the equilibrium system, n is 2.

	y_1	w_1	μ_1^y	μ_1^w	y_2	w_2	μ_2^y	μ_2^w	q	s
y_1	$-D_{11}c_1 - D_{22}\phi_1$	$-D_{32}\phi_1$	1	0	0	0	0	0	0	$-D_{21}c_1$
w_1	$-D_{23}\phi_1$	$-D_{33}\phi_1$	0	1	0	0	0	0	-1	0
μ_1^y	μ_1^y	0	y_1	0	0	0	0	0	0	0
μ_1^w	0	μ_1^w	0	w_1	0	0	0	0	0	0
y_2	0	0	0	0	$-D_{11}c_2 - D_{22}\phi_2$	$-D_{32}\phi_2$	1	0	0	$-D_{21}c_2$
w_2	0	0	0	0	$-D_{23}\phi_2$	$-D_{33}\phi_2$	0	1	-1	0
μ_2^y	0	0	0	0	μ_2^y	0	y_2	0	0	0
μ_2^w	0	0	0	0	0	μ_2^w	0	w_2	0	0
q	0	1	0	0	0	1	0	0	0	0
s	1	0	0	0	1	0	0	0	0	$-Dg$

We partitioned the Jacobian matrix in blocks and labeled each column by its corresponding variable, and we assigned to each equation its key related variable. For example, y_1 is associated with the first-order conditions on output of the individual optimization problem for agent 1, μ_1^y with the complementary slackness condition of the constraint $y_1 \geqslant 0$ and q with the market-clearing condition. The dependency relationships between variables in the equilibrium system is illustrated by the sparsity pattern of the matrix in figure 2. It may be easier to analyze it in figure 3. The block relevant for temporary equilibrium is the top-left square block of size $4n + 1$. The top-left square block of size $4n$ corresponds to individual optimal behavior.

Lemma 7. *The hessian of the profit function* $(y_i, w_i) \mapsto \pi_i(y_i, w_i)$ *is invertible whenever* $y_i > w_i > 0$.

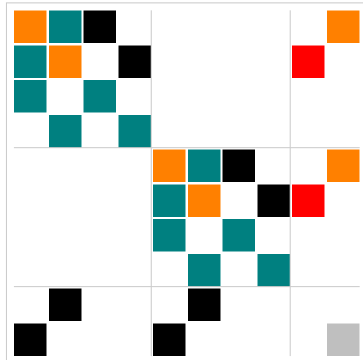
Proof. The hessian of the profit function is given by

$$H_{\pi_i} = \begin{bmatrix} -D_{11}c_i - D_{22}\phi_i & -D_{23}\phi_i \\ -D_{32}\phi_i & -D_{33}\phi_i \end{bmatrix} \tag{25}$$

Its determinant where $y_i > w_i > 0$ is

$$\det H_{\pi_i} = (D_{11}c_i + D_{22}\phi_i)D_{33}\phi_i - (D_{23}\phi_i D_{32}\phi_i)$$

Figure 3: Sparsity pattern of the Jacobian of the equilibrium system, n is 2



As $D_{11}c_i > 0$ when $y_i > 0$, it follows that where $y_i > w_i > 0$

$$\det H_{\pi_i} > D_{22}\phi_i D_{33}\phi_i - (D_{23}\phi_i D_{32}\phi_i) \tag{26}$$

The right-hand side of (26) is the determinant of the hessian of ϕ_i as a function of (y_i, w_i). The convexity of ϕ_i in (y_i, w_i) implies that the hessian on the right-hand side of (26) is positive semidefinite, and thus its determinant is non negative. We conclude that $\det H_{\pi_i} > 0$ whenever $y_i > w_i > 0$, as desired. □

Lemma 8. *Take $p, M, \mathcal{TAC}, s, q$ arbitrarily and let (y_i, w_i) maximize the profits of fisherman i. Then the 4×4 block corresponding to individual optimal behavior in the Jacobian above is invertible if and only if either $y_i > w_i > 0$ or $\mu_i^y, \mu_i^w > 0$.*

Proof. We can write one such 4×4 diagonal block as a block matrix

$$J_i = \begin{bmatrix} H_{\pi_i} & I \\ \operatorname{diag}(\mu_i^y, \mu_i^w) & \operatorname{diag}(y_i, w_i) \end{bmatrix} \tag{27}$$

where each block is 2×2. The matrix H_{π_i} is the hessian of the profit function $(y_i, w_i) \mapsto \pi_i(y_i, w_i)$

$$\begin{bmatrix} -D_{11}c_i - D_{22}\phi_i & -D_{32}\phi_i \\ -D_{23}\phi_i & -D_{33}\phi_i \end{bmatrix}$$

Note that the two bottom blocks of J_i are diagonal matrices and therefore they commute (i.e., their matrix product is the same, irrespective of the order of multiplication). We can thus write the determinant of J_i as

$$\det J_i = \det(H_{\pi_i} \operatorname{diag}(y_i, w_i) - I \operatorname{diag}(\mu_i^y, \mu_i^w))$$

Expanding the first product

$$H_{\pi_i}\,\mathrm{diag}(y_i, w_i) = \begin{bmatrix} y_i(-D_{11}c_i - D_{22}\phi_i) & w_i(-D_{32}\phi_i) \\ y_i(-D_{23}\phi_i) & w_i(-D_{33}\phi_i) \end{bmatrix}$$

and thus

$$\det J_i = \det \begin{bmatrix} y_i(-D_{11}c_i - D_{22}\phi_i) - \mu_i^y & w_i(-D_{32}\phi_i) \\ y_i(-D_{23}\phi_i) & w_i(-D_{33}\phi_i) - \mu_i^w \end{bmatrix}$$

It can now be verified by simple substitutions that the determinant of the 2×2 matrix above is zero if and only if $y_i, w_i > 0$ or $\mu_i^y, \mu_i^w > 0$, which completes the proof. □

Corollary 3. *Consider $p, M, \mathcal{TAC}$ and (y, w, q, s) such that y_i, w_i maximizes profits for every fisherman i. The Jacobian of the individual optimality subsystem (8–11) is invertible if and only if for all agents $i \in \mathcal{I}$ either $y_i, w_i > 0$ or $\mu_i^y, \mu_i^w > 0$.*

Proof. The block corresponding to individual optimal behavior is a block-diagonal matrix with n 4×4 diagonal elements coming from the four equations and four variables involved in the first-order conditions of problem (4). Therefore, this whole $4n \times 4n$ block is invertible if and only if each of the diagonal blocks is invertible. The conditions for invertibility are proved in the preceding Lemma. □

Lemma 9. *The Jacobian of the top-left square block of size $4n + 1$ of the Jacobian in (2), referring to the temporary equilibrium equations, is invertible whenever $y_i > w_i > 0$ for all $i \in \mathcal{I}$.*

Proof. We can write the temporary equilibrium block as the following block matrix:

$$J = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{28}$$

Block A corresponds to the top-left square block of size $4n$ corresponding to the individual optimality conditions. B is $4n \times 1$ C is $1 \times 4n$, and D is 1×1.

As we already know from Corollary 3 that A is invertible, we can write the determinant of the matrix J in (28) as

$$\det(J) = \det(A)\det(D - CA^{-1}B)$$

Because $D = 0$, the invertibility of J hinges on the invertibility of $CA^{-1}B$.

Note that because of the block-diagonal structure of A, we can restrict ourselves without loss of generality to the case $n = 1$ displayed in (29).

$$\begin{bmatrix} y_1 & -D_{11}c_1 - D_{22}\phi_1 & -D_{32}\phi_1 & 1 & 0 & 0 & -D_{21}c_1 \\ w_1 & -D_{23}\phi_1 & -D_{33}\phi_1 & 0 & 1 & -1 & 0 \\ \mu_1^y & \mu_1^y & 0 & y_1 & 0 & 0 & 0 \\ \mu_1^w & 0 & \mu_1^w & 0 & w_1 & 0 & 0 \\ q & 0 & 1 & 0 & 0 & 0 & 0 \\ s & 1 & 0 & 0 & 0 & 0 & -Dg \end{bmatrix} \tag{29}$$

In that case, $CA^{-1}B$ is a 1×1 matrix, and some calculation shows that its value is

$$\frac{D_{11}c_i + D_{22}\phi_i}{\det H_{\pi_i}}$$

which is always strictly negative by Assumption. Therefore, J is invertible, as desired. □

Lemma 10. *Let $\Gamma \subset \mathbb{R}^3$ be the set of tuples $p, M, \mathcal{TAC}$ where agents violate quota in equilibrium and the Jacobian of the equilibrium system (2) is not invertible. Then Γ is a nowhere dense set of measure zero.*

Proof. Graphically, invertibility of the Jacobian corresponds to the temporary equilibrium curve $s \mapsto Y(s)$ being *transversal* to the stock growth curve $s \mapsto g(s)$. It may be the case that those curves are tangent, and therefore we cannot prove that the Jacobian (2) of the equilibrium system will be invertible for *all* $(p, M, \mathcal{TAC})$. However, we can use the transversality Theorem from Appendix C to show that this will happen only in a nowhere dense set of measure zero.

To that end, substitute the column corresponding to the derivatives with respect to s with a column with derivatives with respect to the $\mathcal{TAC}$ to obtain the following square matrix (again, the case $n = 1$ is sufficient)

$$\begin{bmatrix} y_1 & -D_{11}c_1 - D_{22}\phi_1 & -D_{32}\phi_1 & 1 & 0 & 0 & 0 \\ w_1 & -D_{23}\phi_1 & -D_{33}\phi_1 & 0 & 1 & -1 & 0 \\ \mu_1^y & \mu_1^y & 0 & y_1 & 0 & 0 & 0 \\ \mu_1^w & 0 & \mu_1^w & 0 & w_1 & 0 & 0 \\ q & 0 & 1 & 0 & 0 & 0 & 0 \\ s & 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{30}$$

Again, partitioning the matrix in (30) in blocks A, B, C, D as in (28), with A being the $4n \times 4n$ individual optimality part (which is invertible), we can write its determinant as

$$\det(A)\det(D - CA^{-1}B)$$

Because we know A is invertible, the invertibility of the Jacobian matrix above hinges on the invertibility of $D - CA^{-1}B$. We can show that

$$D - CA^{-1}B = \begin{bmatrix} \frac{-D_{11}c_i - D_{22}\phi_i}{\det H_{\pi_i}} & -1 \\ \frac{-D_{32}\phi_i}{\det H_{\pi_i}} & 0 \end{bmatrix} \tag{31}$$

It is clear that the determinant of the matrix in (31) is not zero. Therefore, the matrix in (30) is invertible.

The result follows then from the transversality Theorem. See Theorem 11 in Appendix C. □

B Nonsmooth Optimality Conditions when Violations Are Zero

Suppose q, s are such that there are optimal y_i, w_i for agent i where $y_i = w_i$. At this point, the map $(y_i, w_i) \mapsto \phi^+(M, y_i, w_i)$ need not be differentiable. The first-order conditions at this point are

$$0 \in \partial_{y_i,w_i} L \tag{32}$$

where $\partial_{y_i,w_i} L$ is the subgradient of the Lagrangian

$$L = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i) - \phi^+(M, y_i, w_i) + \mu_i^y y_i + \mu_i^w w_i$$

So (32) translates into

$$\begin{aligned} p - D_1 c_i + \mu_i^y &= \eta \\ -q + \mu_i^w &= \nu \end{aligned} \tag{33}$$

for some $(\eta, \nu) \in \partial_{y_i,w_i} \phi^+$. As $\phi^+ = \max\{\phi, 0\}$ and ϕ is convex and differentiable, we can write the subgradient $\partial_{y_i,w_i} \phi^+$ as the convex combination of $\nabla_{y_i,w_i} \phi$ and $(0, 0)$. Therefore, we can state the following.

Lemma 11. *Fix $M, \mathcal{TAC}, q, s$. If $y_i, w_i \geqslant 0$ with $y_i = w_i$ maximizes profits for i, then there exists $\alpha_i \in [0, 1]$, $\mu_i^y \geqslant 0$, and $\mu_i^w \geqslant 0$ such that $y_i \mu_i^y = 0$, $w_i \mu_i^w = 0$ and*

$$\begin{aligned} p - D_1 c(y_i, s, \theta_i) + \mu_i^y &= \alpha_i D_2 \phi(M, y_i, w_i) \\ -q + \mu_i^w &= \alpha_i D_3 \phi(M, y_i, w_i) \end{aligned}$$

We can now examine who will have positive production and who will hold positive amounts of quota.

Lemma 12. *Fix $M, \mathcal{TAC}, q, s$. If $p > D_1 c(0, s, \theta_i)$ for some fishermen i, and (y_i, w_i) maximizes profits, then $y_i > 0$. Conversely, if there is a profit-maximizing (y_i, w_i) with $y_i > 0$, then $p \geqslant D_1 c(0, s, \theta_i)$, with strict inequality if $q > 0$.*

Proof. Straight out of the first order conditions. See Lemma 11 at page 30. □

Lemma 13. *Fix $M, \mathcal{TAC}, q, s, y_i > 0$. If $q < D_3\phi(M, y_i, 0)$ for some fishermen i, and (y_i, w_i) maximizes profits, then $w_i > 0$. Conversely, if there is a profit-maximizing (y_i, w_i) with $w_i > 0$, then $q < -D_3\phi(M, y_i, 0)$.*

Proof. Follows from the first-order conditions and the fact that ϕ is strictily convex in w_i. □

C Transversality Theory

Some of our results rely on a collection of propositions loosely referred to as "the transversality Theorem(s)". These results formalize and generalize the heuristic analysis of the solution set of a system of nonlinear equations through "counting equations and unknowns" by stating that smooth curves and surfaces (more generally, manifolds) are generally transversal. It is thus necessary to be precise about the definitions of "smooth", "manifold", "generic" and "transversal". These concepts have had useful applications in economic theory for a long time (see Mas-Colell (1985)). We provide a brief introduction to these topics in this Appendix. See Guillemin and Pollack (2010) or Hirsch (1976) for a textbook treatment of the subject. The Theorems listed here were taken from Aubin and Ekeland (2006).

Let $X \subset \mathbb{R}^k$ and $Y \subset \mathbb{R}^l$ be arbitrary. We say a map $f : X \to Y$ is ***smooth map*** of class C^r if for each $x \in X$ there is an open set $U \subset X$ about x and a map $F : U \to \mathbb{R}^l$ of class C^r, $r \geqslant 1$ such that F coincides with f on $U \cap X$. We call f a ***diffeomorphism*** if it is a smooth bijection with a smooth inverse. *Examples of smooth maps and diffeomorphisms*: the identity map is always smooth, but the map $x \mapsto x^3$ of $(-1, 1)$ on itself is not a diffeomorphism; it is smooth with a continuous inverse, but the inverse $y \mapsto y^{1/3}$ is not differentiable at $y = 0$.

A set $X \subset \mathbb{R}^n$ is a m-dimensional ***smooth manifold*** if every point $x \in X$ has a neighborhood in X that is diffeomorphic to an open subset of $\mathbb{R}^m$. *Examples of smooth manifolds*: any singleton is a 0-dimensional smooth manifold; any open set in $\mathbb{R}^k$ is a k-dimensional manifold; the graph of any smooth function $f : \mathbb{R}^k \to \mathbb{R}$ is a smooth manifold of dimension $k - 1$.

Let x be an element of an m-dimensional manifold $M \subset \mathbb{R}^n$ Let $U \subset \mathbb{R}^m$ be an open set containing x, and $g : U \to M$ the smooth parametrization of a neighborhood of x. The ***tangent space*** at x relative to M, denoted by T_xM, is the image of the linear operator $Dg(x) : \mathbb{R}^m \to \mathbb{R}^n$. One can prove that the tangent space does not depend on the choice of parametrization g. *Examples of tangent spaces*: for every $x \in \mathbb{R}^n$ we have $T_x\mathbb{R}^n = \mathbb{R}^n$; relative to the 2-dimensional unit-sphere in $\mathbb{R}^3$, the tangent space at any point is $\mathbb{R}^2$. One can show that the tangent space has the same dimension as the manifold it is tangent to.

Consider two smooth manifolds $M \subset \mathbb{R}^k$ and $N \subset \mathbb{R}^l$ and a smooth map $f : M \to N$ with $f(x) = y$. The ***derivative*** $Df(x) : T_xM \to T_yN$ is defined as follows. Since f is smooth, there exists an open set about x and a smooth map $F : W \to \mathbb{R}^l$ that coincides with f on $W \cap M$. For all $v \in T_xM$ define the $Df(x) \cdot v$ to be equal to the directional derivative $DF(x) \cdot v$. One can prove that the derivative of f at x does not depend on the choice extension F.

Let us now move on to the notion of *genericity*. Let X be a complete metric space. A G_δ subset of X is defined as the intersection of a countable family of open subsets of X. We say $Y \subset X$ is ***generic set*** if it contains a dense G_δ of X. We say a statement $P(x)$ about points $x \in X$ is a ***generic property*** if the set $\{x \in X : P(x) \text{ is true }\}$ is generic.

Finally, let us define the notion of *transversality*. It is a generalization of the notion of *regularity*. Let $f : X \to Y$ be a smooth map between smooth manifolds and Z be a submanifold of Y such that $Z \cap f(X) \neq \emptyset$. It may or may not be the case that $f^{-1}(Z)$ is a smooth submanifold of X. A sufficient condition for that is that f be *transversal* to Z in a sense that we explain now in increasing level of generality.

- Let $X = Y = \mathbb{R}^n$, $Z = \{0\}$. In this case, $f(x) \in Z$ represents a *square system of nonlinear equations*. We say f is transversal to Z if for all $x \in f^{-1}(Z)$ the derivative $Df(x)$ is *invertible*. The *inverse function Theorem* guarantees that $f^{-1}(Z)$ is a set of isolated points, or, in other words, it is a manifold of dimension 0 (equivalently, with the same codimension of Z in Y: n).

- Let $X = \mathbb{R}^n, Y = \mathbb{R}^p$, $Z = \{0\}$. In this case, $f(x) \in Z$ represents a *nonlinear system of equations* We say f is transversal to Z if for all $x \in f^{-1}(Z)$ the derivative $Df(x)$ is *surjective* (synonym: onto). This is the same as saying that 0 is a ***regular value*** of f. The *implicit function Theorem* guarantees that $f^{-1}(Z)$ is a smooth manifold of dimension $n - p$ (equivalently, with the codimension of Z in Y: p).

- Let $X = \mathbb{R}^n, Y = \mathbb{R}^p$, and Z some m-dimensional submanifold of Y. In this case, $f(x) \in Z$ represents a *system of nonlinear inclusions*. We say f is transversal to Z if for all $x \in X$:

  $$Im(Df(x)) + T_{f(x)}(Z) = \mathbb{R}^p$$

  The *implicit function Theorem* will guarantee that $f^{-1}(Z)$ is a smooth manifold of dimension $n - p + m$ (equivalently, with the codimension of Z in Y: $p - m$).

- General case, where X, Y and $Z \subset Y$ are manifolds. In this case, $f(x) \in Z$ represents a *system of nonlinear inclusions*. We say f is ***transversal to*** Z if for all $x \in X$:

  $$Im(Df(x)) + T_{f(x)}(Z) = T_{f(x)}Y$$

The *implicit function Theorem* guarantees that $f^{-1}(Z)$ is a smooth manifold with codimension equal to the codimension of Z in Y.

in the nonlinear system $f(x) \in Z$. The dimension of $f^{-1}(Z)$ is the formalization of the idea of "number of degrees of freedom" in the nonlinear system $f(x) \in Z$.

We can now state versions of the transversality Theorem that are sufficiently general for our needs. In the following, U is an open subset of $\mathbb{R}^n$, Λ is a separable Banach space and Z is a C^∞ submanifold of $\mathbb{R}^p$ with codimension q.

Theorem 10 (Transversality Theorem 1)**.** *Let $f : U \times \Lambda \to \mathbb{R}^p$ be a smooth map of class C^r, $r \geqslant 1$. If f is transversal to Z and $r \geqslant \max\{1, n-q+1\}$ then*

$$L = \{\lambda \in \Lambda : x \mapsto f(x, \lambda) \textit{ is transversal to } Z\}$$

is a generic set in Λ.

The fact that V is infinite dimensional allows us to pick the function itself as a parameter.

Corollary 4 (Transversality Theorem 2)**.** *The property*

$$P(f) = \{f : U \to \mathbb{R}^p \textit{ is transversal to } Z\}$$

is generic in $C^r(U; \mathbb{R})$.

In particular, this Corollary implies that "for most" (that is, generic) smooth, square, nonlinear systems of equations, the solution set is a set of isolated points (a 0-dimensional manifold). More generally, if this square system of equations has k (exogenous) parameters then "for most" (that is, generic) smooth, nonlinear systems of equations, the solution set as a function of the exogenous parameters is a manifold of dimension k.

Let us quickly relate the transversality Theorem to the heuristic analysis of the solution set via "numbers of variables vs. numbers of equations" arguments that is familiar from linear algebra. The dimension of the solution set $f^{-1}(Z)$ is the formal concept of "degrees of freedom"; The codimension of the solution set $f^{-1}(Z)$ "typically" is the number of equations. The transversality Theorem formalizes, generalizes and generically validates the following heuristic *local* analysis of systems of nonlinear equations:

- degrees of freedom = number of variables - number of equations;
- positive degrees of freedom imply multiple solutions;
- negative degrees of freedom imply no solutions (because the empty set is the only manifold of negative dimension);

- zero degrees of freedom imply a unique solution.

If we confine ourselves to a finite-dimensional set of parameters Λ, we can strengthen the conclusion of the transversality Theorem.

Theorem 11 (Transversality Theorem 3)**.** *Let $M \subset \mathbb{R}^n$, $\Lambda \subset \mathbb{R}^l$, and $Z \subset \mathbb{R}^p$ be smooth manifolds. Let $f : M \times \Lambda \to \mathbb{R}^p$ be a smooth map. If f is transversal to Z then*

$$L = \{\lambda \in \Lambda : x \mapsto f(x, \lambda) \text{ is transversal to } Z\}$$

is a generic set in Λ, and $\Lambda \setminus L$ has measure zero.

D Assumptions on the Monitoring Function

We now make some additional Assumptions based on the interpretation of $\phi(M, y_i, w_i)$ as the expected fine for violation. First, we assume ϕ to be jointly convex in (y_i, w_i), but not necessarily strictly so, as that rules out interesting violation measures like $v(y_i, w_i) = (y_i - w_i)/(1 + w_i)$. Second, we also assume that $|D_2\phi| \leqslant |D_3\phi|$ That means that given any change in catch y_i, there is a (weakly) smaller change in quota holdings w_i that changes the total fine at least as much as the change in y_i. Third, we assume that for all $(y_i, w_i) \neq (y_i, \widetilde{y}_i)$ we have

$$(D_2\phi(M, y_i, w_i) - D_2\phi(M, \widetilde{y}_i, \widetilde{w}_i))\,(D_3\phi(M, \widetilde{y}_i, \widetilde{w}_i) - D_3\phi(M, y_i, w_i)) > 0$$

That simply means that if the marginal fines $D_2\phi$ go up, then so should the marginal fine savings from buying quota $-D_3\phi$, and vice-versa. Finally, note that it follows from our Assumptions that if $D_2\phi$ and $D_3\phi$ exist at a point where $y_i = w_i$, then $D_2\phi = D_3\phi$ at that point. This equality may or may not hold at other points depending on the violation measure (for example, it will always hold if violations are measured absolutely).

Note that it may well be the case that ϕ^+ is not differentiable when violations are exactly zero, that is, when $y_i = w_i$. Example: $\phi(M, y_i, w_i) = \rho(M)((y_i - w_i) + (y_i - w_i)^2)$. Allowing this type of nonsmoothness at zero violations makes the analysis of first-order conditions a little more complex but it enriches the model in a way that we believe is significant: it makes it possible for respecting one's quota to be an optimal action.

References

Aubin, J.-P., and I. Ekeland (2006): *Applied Nonlinear Analysis*. Dover Publications.

Bonzon, K., K. McIlwain, C. K. Strauss, and T. Van Leuvan (2010): "Catch Share Design Manual," Discussion paper, Environmental Defense Fund.

Chavez, C., and H. Salgado (2005): "Individual Transferable Quota Markets under Illegal Fishing," *Environmental and Resource Economics*, 31(3), 303–324.

Clark, C. W. (1973): "The Economics of Overexploitation," *Science*, 181(4100), 630–634.

——— (2010): *Mathematical Bioeconomics: The Mathematics of Conservation*. Wiley, 3 edn.

Conrad, J. M. (2010): *Resource Economics*. Cambridge University Press, 2 edn.

Copes, P. (1986): "A critical review of the individual quota as a device in fisheries management," *Land economics*, 62(3), 278–291.

Costello, C., S. D. Gaines, and J. Lynham (2008): "Can Catch Shares Prevent Fisheries Collapse?," *Science*, 321(5896), 1678–1681.

Dales, J. H. (1968): *Pollution, property & prices*. University of Toronto P.

Debreu, G. (1970): "Economies with a Finite Set of Equilibria," *Econometrica*, 38(3), 387–392.

Freeman, J., and C. D. Kolstad (2007): *Moving to markets in environmental regulation: lessons from twenty years of experience*. Oxford University Press.

Grafton, R. Q., R. Arnason, T. Bjorndal, D. Campbell, H. F. Campbell, C. W. Clark, R. Connor, D. P. Dupont, R. Hannesson, R. Hilborn, J. E. Kirkley, T. Kompas, D. E. Lane, G. R. Munro, S. Pascoe, D. Squires, S. I. Steinshamn, B. R. Turris, and Q. Weninger (2006): "Incentive-based approaches to sustainable fisheries," *Canadian Journal of Fisheries and Aquatic Sciences*, 63(3), 699–710.

Grafton, R. Q., T. Kompas, and R. W. Hilborn (2007): "Economics of Overexploitation Revisited," *Science*, 318(5856), 1601–1601.

Guillemin, V., and A. Pollack (2010): *Differential Topology*. American Mathematical Society, reprint edn.

Hatcher, A. (2005): "Non-compliance and the quota price in an ITQ fishery," *Journal of Environmental Economics and Management*, 49(3), 427–436.

Hirsch, M. W. (1976): *Differential Topology*. Springer.

Knapp, G., and J. J. Murphy (2010): "Voluntary Approaches to Transitioning from Competitive Fisheries to Rights-Based Management: Bringing the Field into the Lab," *Agricultural and Resource Economics Review*, 39(2), 245–261.

Malik, A. S. (1990): "Markets for pollution control when firms are noncompliant," *Journal of Environmental Economics and Management*, 18(2, Part 1), 97–106.

Mas-Colell, A. (1985): *The Theory of General Economic Equilibrium: A Differentiable Approach*. Cambridge University Press, New York.

Moloney, D. G., and P. H. Pearse (1979): "Quantitative Rights as an Instrument for Regulating Commercial Fisheries," *Journal of the Fisheries Research Board of Canada*, 36(7), 859–866.

Montero, J.-P. (2004): "Tradable Permits with Incomplete Monitoring: Evidence from Santiago's Particulate Permits Program," Cambridge Working Papers in Economics 0457, Faculty of Economics, University of Cambridge.

Montgomery, W. (1972): "Markets in licenses and efficient pollution control programs," *Journal of Economic Theory*, 5(3), 395–418.

Perman, R., Y. Ma, M. Common, D. Maddison, and J. Mcgilvray (2012): *Natural Resource and Environmental Economics*. Prentice Hall, 4 edn.

Shannon, C. (2008): "Determinacy and Indeterminacy of Equilibria," in *The New Palgrave Dictionary of Economics*, ed. by S. N. Durlauf, and L. E. Blume. Palgrave Macmillan, 2 edn.

Tietenberg, T., and L. Lewis (2008): *Environmental & Natural Resource Economics*. Addison Wesley, 8 edn.

Worm, B., E. B. Barbier, N. Beaumont, J. E. Duffy, C. Folke, B. S. Halpern, J. B. C. Jackson, H. K. Lotze, F. Micheli, S. R. Palumbi, E. Sala, K. A. Selkoe, J. J. Stachowicz, and R. Watson (2006): "Impacts of Biodiversity Loss on Ocean Ecosystem Services," *Science*, 314(5800), 787–790.