Ideas for Math Education: a Two-Part Talk

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Part 1:

The Khan Academy

What is the Khan Academy?

- A website with short (10-20min) video tutorials about many school topics. Right now, mostly focused on math. All math from Kindergarden from High School is covered!

- The website also allows students to solve exercises online. The website has an algorithm that generates new problems automatically, and recommends different level of exercises based on the success rate of the student with the current exercises. The student can create an account to track his progress.

- Tutors can register to track their student's progress.

- You actually do <u>not</u> need internet access to use it.

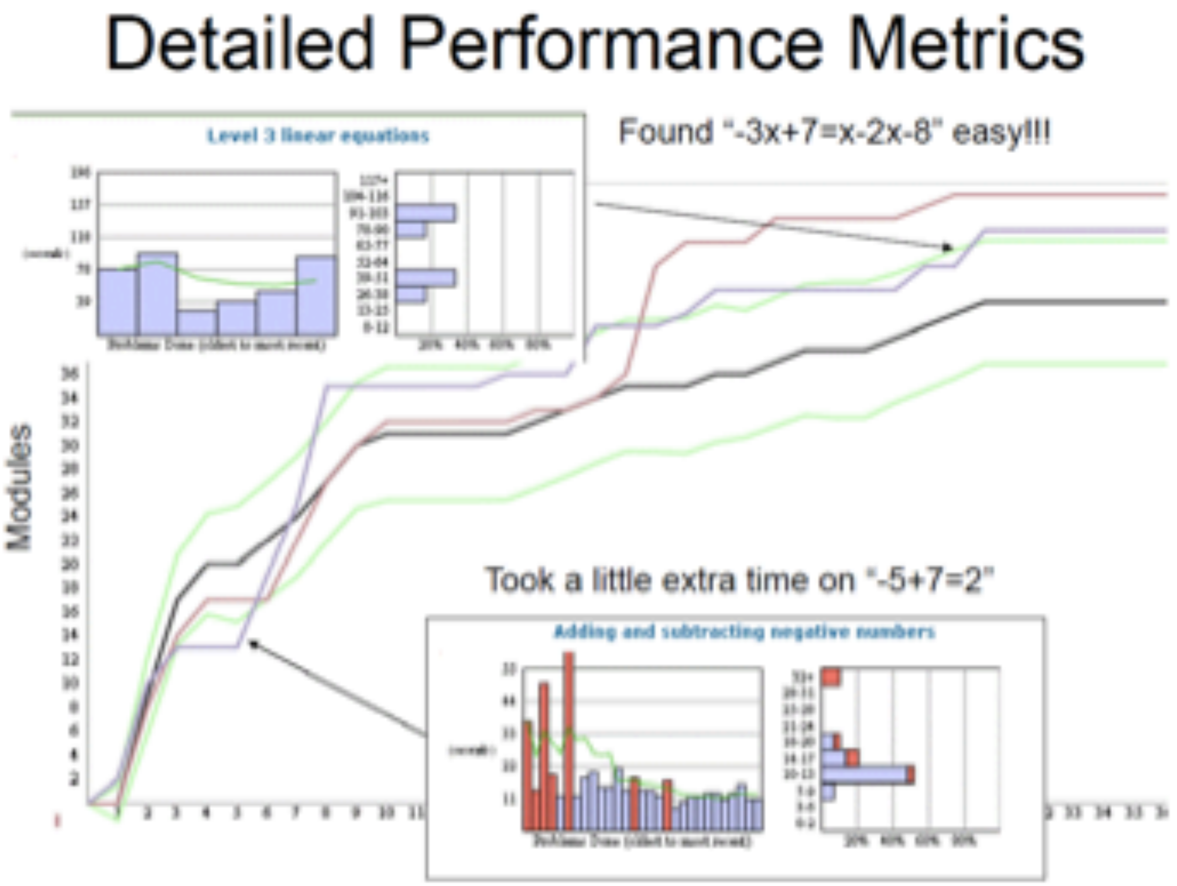
Quick video about Salman Khan and his Academy

(Use first video on the "About Us" section of khanacademy.org if internet is working.)

Quick Demonstration of the Interface

(If an internet connection is available. Don't forget to show the coaching tools.)

This next chart shows the average (black line) progress of a cohort of 30 rising 8th graders on the Khan Academy software over a 6-week period. The horizontal axis is "days working on the site" and the vertical axis is "modules completed." The green lines show one standard deviation above and below the mean. This chart exemplifies both the level of data we are capturing and also highlights the importance of individualized, self-paced instruction and real-time assessment. The purple line shows a student who may have been deemed "slow" by traditional assessments because she was more than one standard deviation below the class average after working on the site for a few days. The reality is that she just needed more time ramping up on negative numbers than the other students in the cohort. Once she was given the chance to become proficient on that concept, she raced forward and ended up being one of th top students in the cohort.



Mr. Khan,

No teacher has ever done me any good--this may sound harsh but I mean it quite literally. I was force fed medication to keep me from talking and chastised for not speaking out when called on. Where I am from blacks are not welcomed with open arms into schools--my mother and her sisters had to go to a small shack two hours from home when they went to school. About five years ago my family collected enough money to move from where i was born, so that I could have a chance at having an education and living a real life. But without a real mastery of elementary math I was slow to progress.

I am now in college and learning more than I ever have in my life. **But an inadequate math background has been holding me back**. I found the Kahn Academy in June of 2009, right after I completed Math 141 (a college algebra course). **I have spent the entire summer on your youtube page.** And I just wanted to thank you for everything you are doing. You are a Godsend. Last week I tested for a math placement exam and I am now in Honors Math 200. **No question was answered incorrectly.** My placement test holder was so impressed by the breadth of my knowledge of math that he said I should be in Linear algebra.

Mr. Khan, I can say without any doubt that you have changed my life and the lives of everyone in my family.

I wish you and the Khan Academy the best of luck,

Part II:

A Discussion on Trial and Error

How do we solve 13x = 234?

Easy! Long Division (*easy?)*:

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```
     18
   ______
13 | 234
    -13
    ---
     104
```

→ **Solution: x = 18.**

How do we solve 13x = 234?

Easy! Long Division (*easy?*):

$$\begin{array}{r|l} & \;\;18 \\ \hline 13 & 234 \\ & -13 \\ \hline & \;\;104 \end{array} \longrightarrow$$

Solution: x = 18.

Is that the best way?

Probably, if you have pen and paper...

Is there another way?

Yes! The oldest method of all!

Solving $13x = 234$ by trial and error:

Solving 13x = 234 by trial and error:

x = 10?

130 ↑

Solving 13x = 234 by trial and error:

x = 10? *130* ↑

x = 20? *260* ↓

Solving 13x = 234 by trial and error:

x = 10? *130* ↑

x = 20? *260* ↓

x = 15? *195* ↑

Solving 13x = 234 by trial and error:

x = 10? *130* ↑

x = 20? *260* ↓

x = 15? *195* ↑

x = 17? *221* ↑

Solving 13x = 234 by trial and error:

x = 10?

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x = 20?

260 ↓

x = 15?

195 ↑

x = 17?

221 ↑

x = 18!

Yes, 234!

Another way to visualize it:

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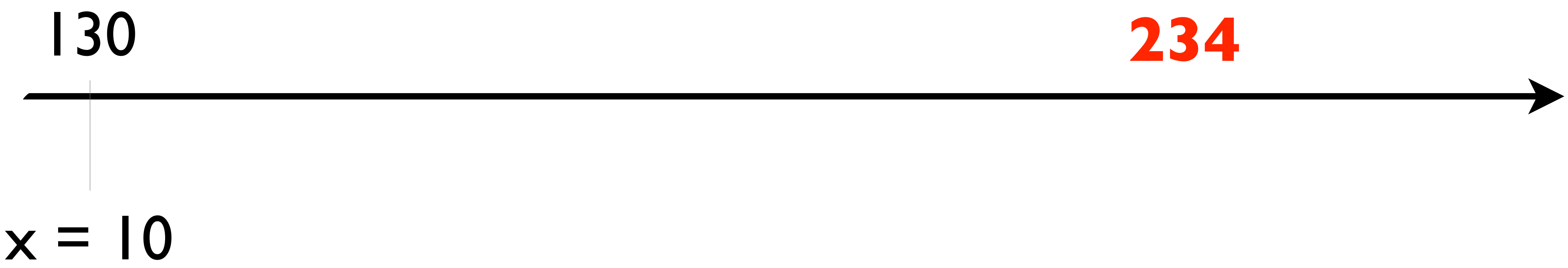

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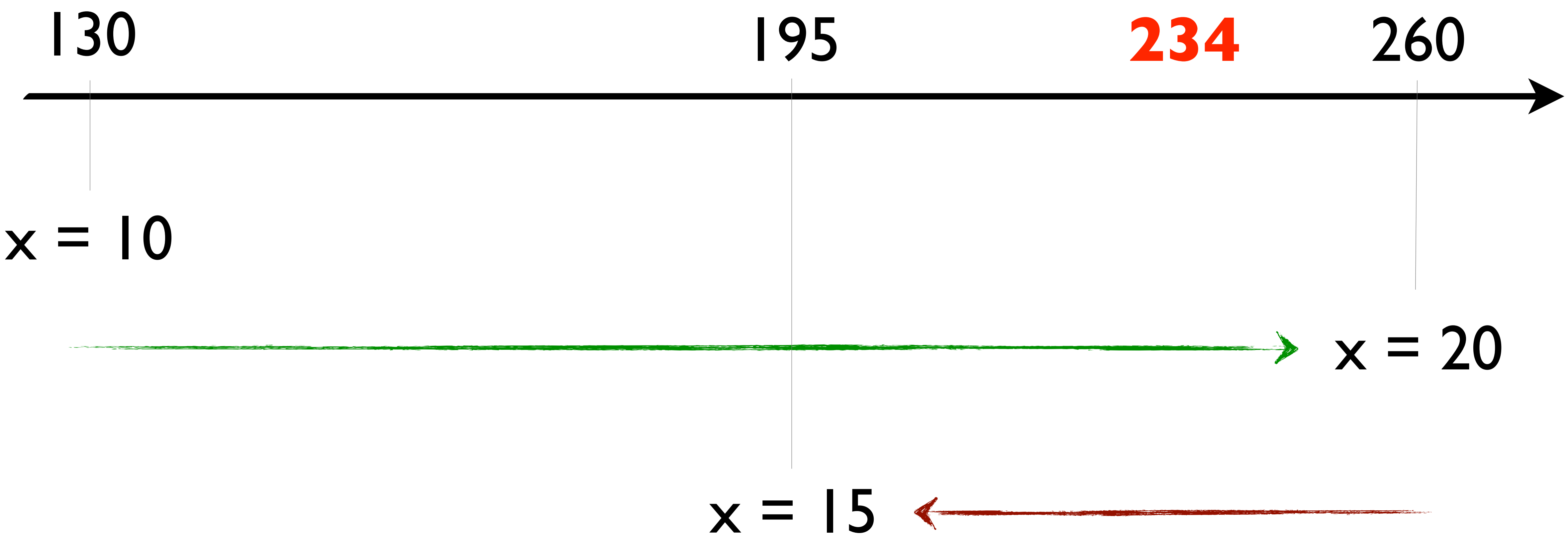

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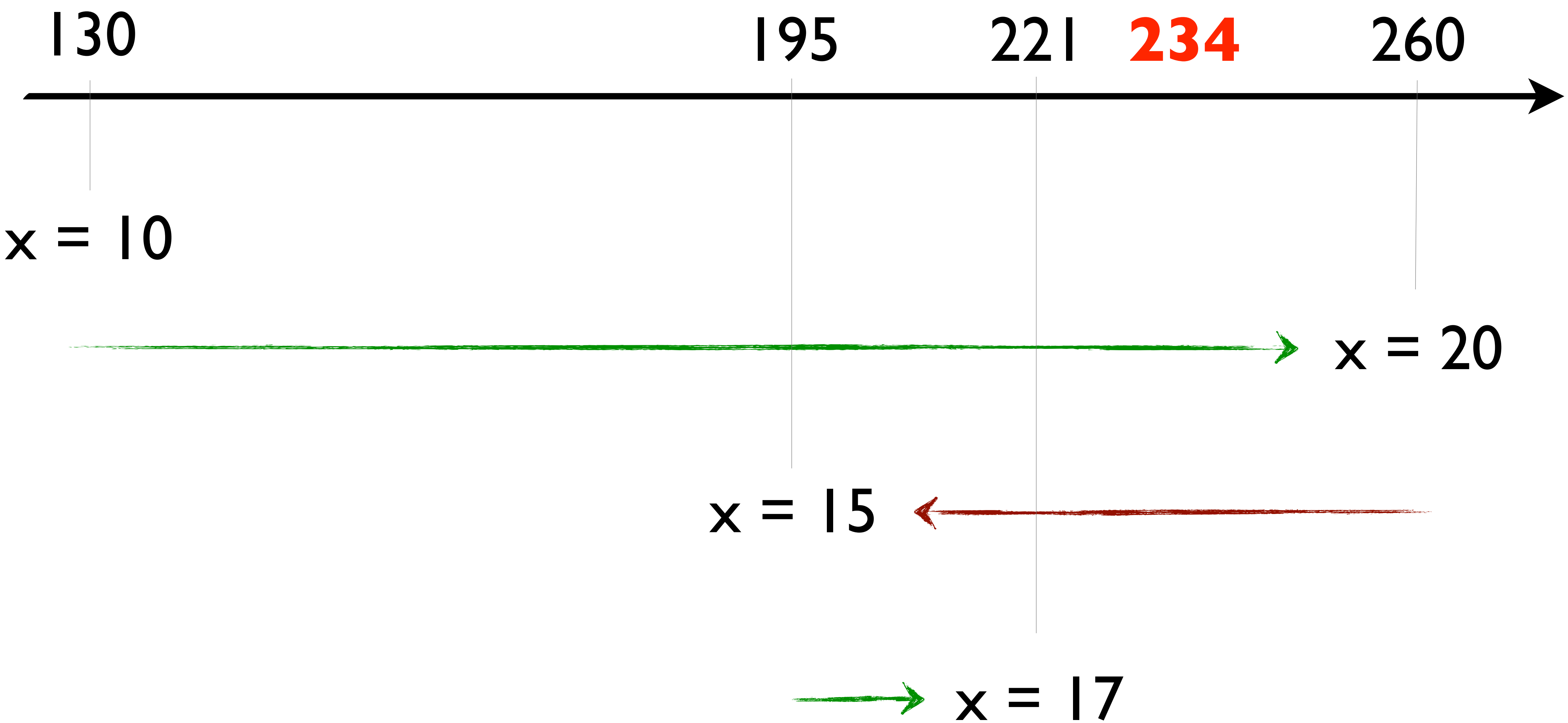

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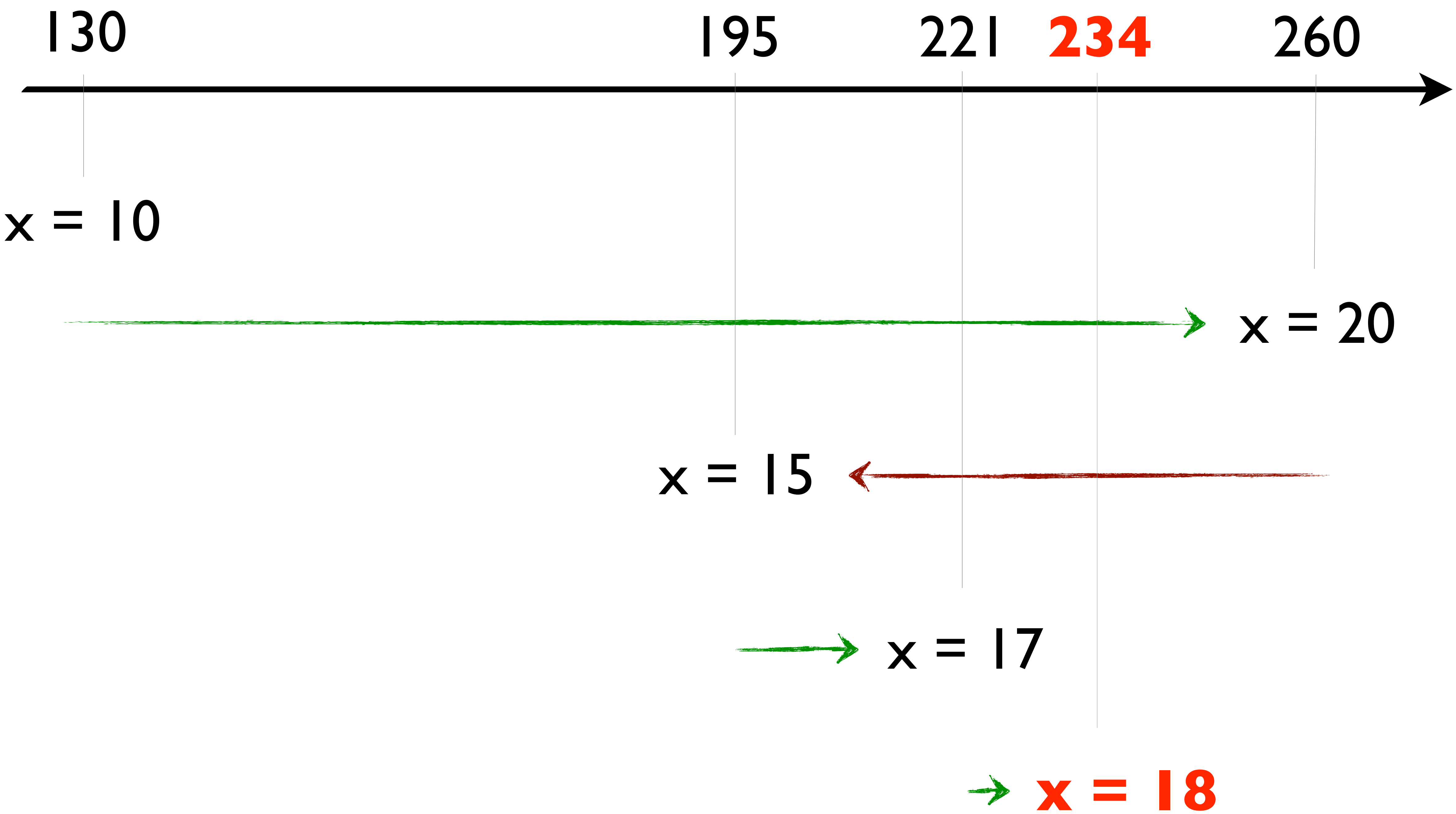

What are the benefits of practicing trial-and-error solutions for our students?

- They see that it is ok to not know how to get the solution for a problem right away. It's *normal*, and it is no excuse for not *giving it a shot.*

- It practices their estimation skills. It is important to be able to *quickly* make *educated* guesses and *rough estimates*!

- It is not a cookbook recipe. They get to *make their own decisions* in the process. They are *in control.* *Note:* some guesses are better than others, and there are ways to "minimize" the number of guesses (we'll talk about bisection).

What are the benefits of practicing trial-and-error solutions for our students?

- The mechanics of the method are very clear. It is almost "common sense applied to a drawing". Compare with long division!

- It gives them an appreciation for the speed, precision and elegance of long division later.

- It beats long division if you do not have pen and paper! Less things to remember at each step!

- It stimulates students to be fast at multiplication!*

Some longer-term benefits, or for advanced students:

- It transmits two very important general ideas: that of *stepwise refinement* and (implicitly) that of *approximate or "good enough" solutions*. More complicated problems require that approach, in real life or in math! ($\sqrt{2}$ anyone?)

- It permits the tutor to introduce a beautiful and simple idea that works well for many problems: *bisection*. It is a safe bet once you have bracketed the solution. The idea of bisection also provides a natural bridge to the idea of *logarithm*.

- The ideas of solution by trial-and-error and bisection are very general.

A fictitious dialogue between a student and a tutor

More on Bisection

Bisection at work (1)

x = 10?	130	↑
x = 20?	260	↓
x = 15?	195	↑
x = 17?	221	↑
x = 18!	**234**	

x = 0?	0	↑
x = 100?	1300	↓
x = 50?	650	↓
x = 25?	325	↓
x = 13?	169	↑
x = 19?	247	↓
x = 18!	**234**	

Bisection at work (1)

x = 10?	130 ↑	x = 0?	0 ↑
x = 20?	260 ↓	x = 100?	1300 ↓
x = 15?	195 ↑	x = 50?	650 ↓
x = 17?	221 ↑	x = 25?	325 ↓
x = 18!	**234**	x = 13?	169 ↑
		x = 19?	247 ↓
		x = 18!	**234**

Conclusion: a bracket 10 times larger required only two more guesses...

Bisection at work (2)

Consider the following game: suppose I have with me a copy of *The American Heritage Dictionary of the English Language*, 4th edition. At 2112 pages, it is quite a tome! How many times do I need to open it in order to find any given word?

Bisection at work (2)

Consider the following game: suppose I have with me a copy of *The American Heritage Dictionary of the English Language*, 4th edition. At 2112 pages, it is quite a tome! How many times do I need to open it in order to find any given word?

Answer: never more than 12.

The answer to this apparent puzzle lies in the following fact: once a solution has been bracketed, the bisection method finds the solution in no more than *log(n)* where *n* is the size of the bracket, and the *log* is taken on base 2.

$$log(10) \approx 3.32$$
$$log(100) \approx 6.64$$
$$log(2112) \approx 11.04$$

Opportunities provided by the idea of bisection:

- Introduction to logarithms!
- Introduction to the idea of algorithms!
- Introduction to computer programming! Want to calculate $\sqrt{2}$? It is easy with bisection: make guesses and evaluate x^2 - 2 to gauge accuracy.
- Introduction to *limits* (*calculus)*: if you bisect forever and pick a point inside each interval, where does this sequence of points converge to?

Thanks